Competitive Algorithm Design and Practice
Maximum Flow
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Flow

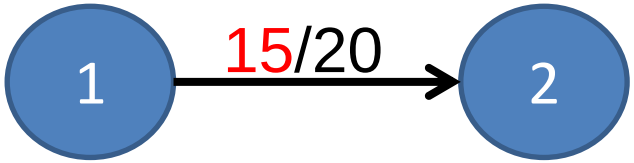
Flow

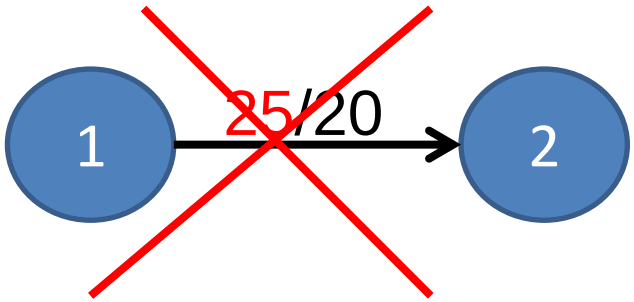
Capacity

pipe

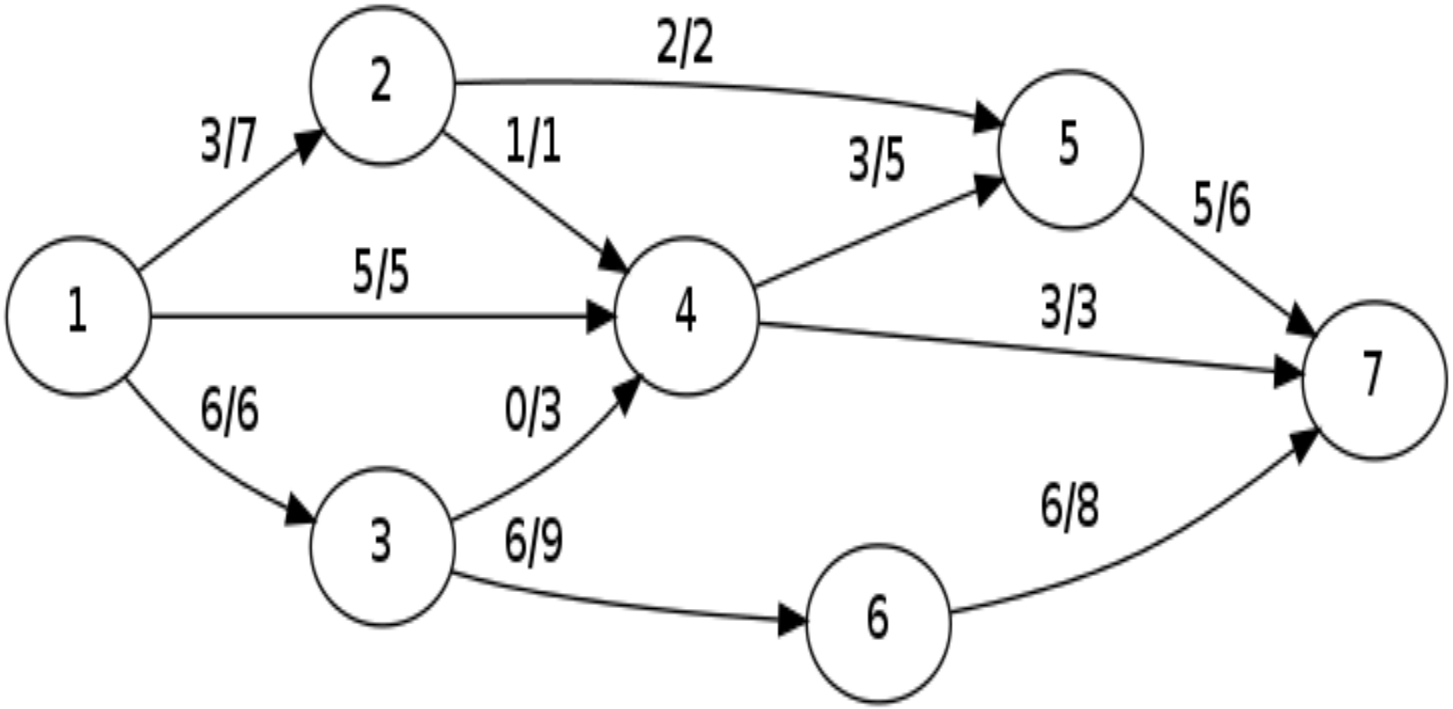
Capacity

$$C(u,v) \geq F(u,v)$$





Flow Network



Flow Network

- **Definition:**
  - A graph used to describe the flow.

Flow Network

- **Information:**

  - capacity

  - flow

- **express:**

  flow / capacity

Flow network

- pipe ：edge (directional or bidirectional)
- capacity of pipe : weight of edge
- pipe joint ：vertex (talk about it later)

Flow network

S-T Flow


M12 东三国
吹田
下新庄
淡路
柴岛
阪急京都线
JR新干线
M13 新大阪
新大阪
M14 西中岛南方
南方
阪急神户线
JR京都线
M15 中津
梅田
T19 中崎町
T18 K11 天神桥筋六丁目
T17 都岛
T16 野江内代
T15 关目高殿
关目成育
T14 千林大宫
T13 大桐丰里
T12 瑞光四丁目
JR神户线
大阪
天满
樱之宫
JR大阪环状线
福岛
M16 梅田
T20 东梅田
南森町
K12 扇町
大阪天满宫
S11 野田阪神
西梅田
Y11
渡边桥
新福岛
北新地
T21 K13
大阪城北诘
京桥
N22
N23 I18 蒲生四丁目
N24 今福鹤见
JR东西线
S12 玉川
Y12 肥后桥
M17 淀屋桥
K14 北滨
T22 天满桥
大阪商务园区
(大阪Business Park)
N21
鴫野
I19 鴫野
JR学研都市线
京桥
大阪城公园
阿波座
本町
C15 S13
Y13
C16
M18
堺筋本町
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S-T Flow


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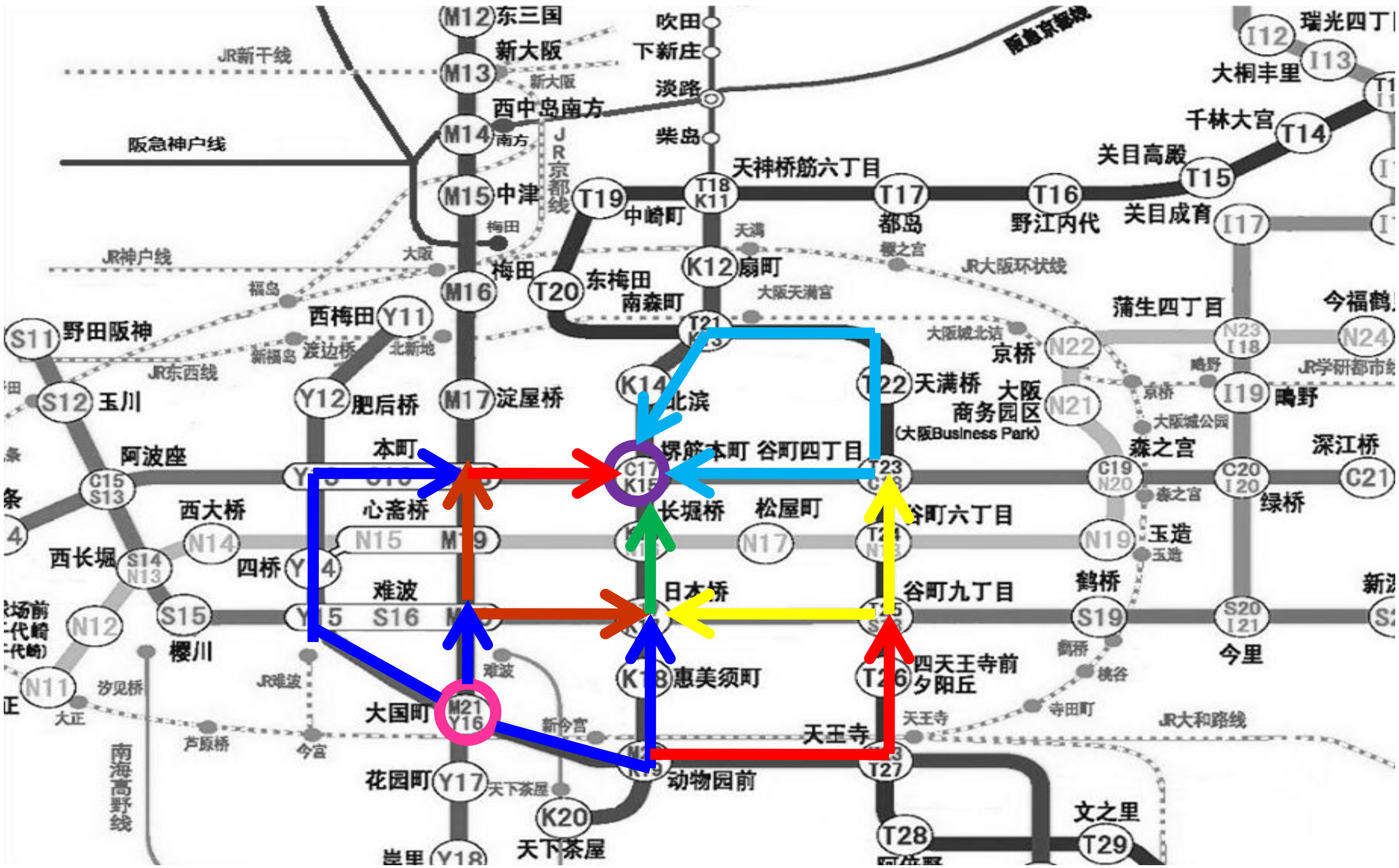
S-T Flow



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JR难波
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芦原桥
新今宫

S-T Flow

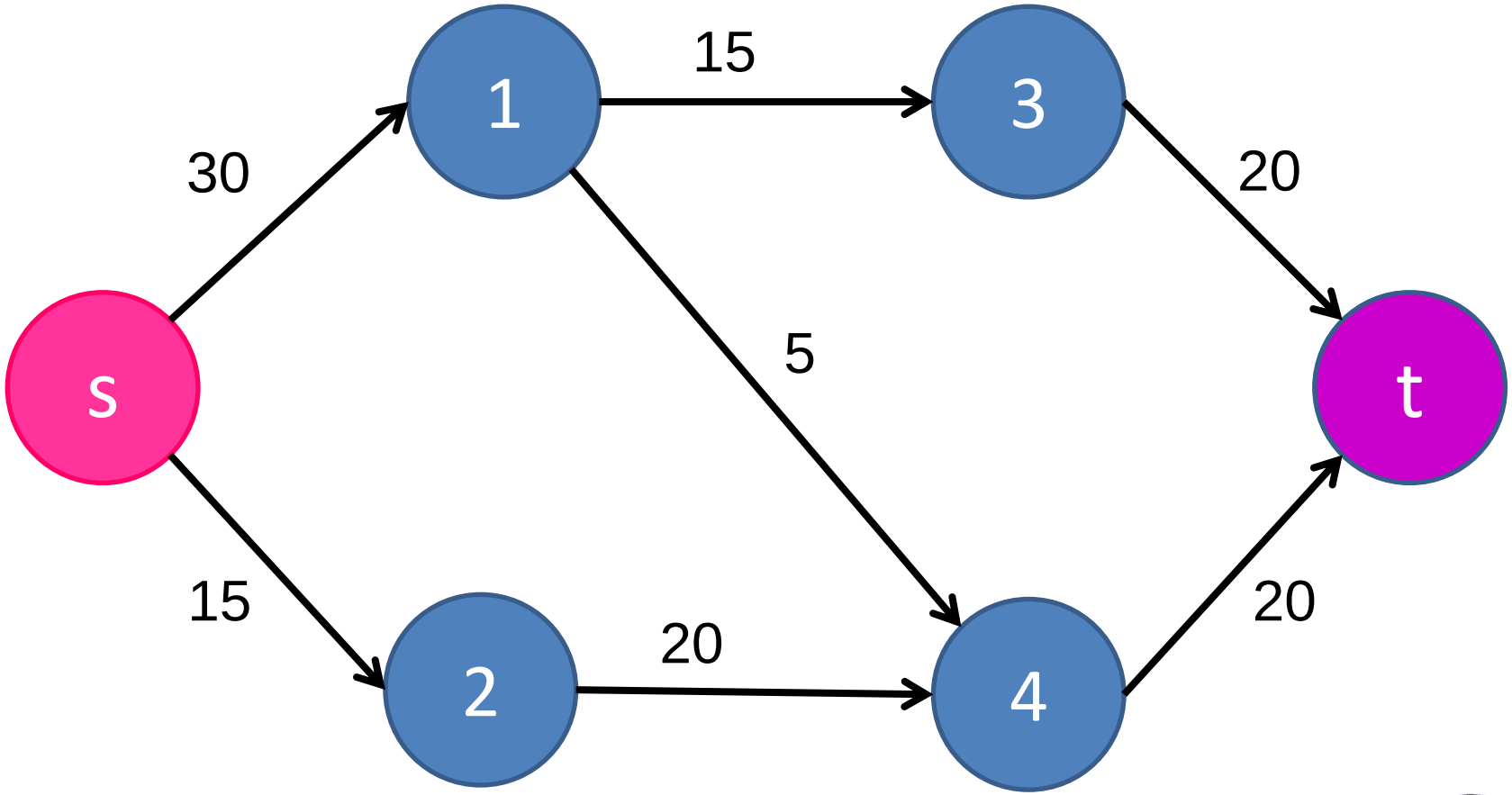
- Two vertice:
  - source **(S)**
  - sink **(T)**

S-T Flow

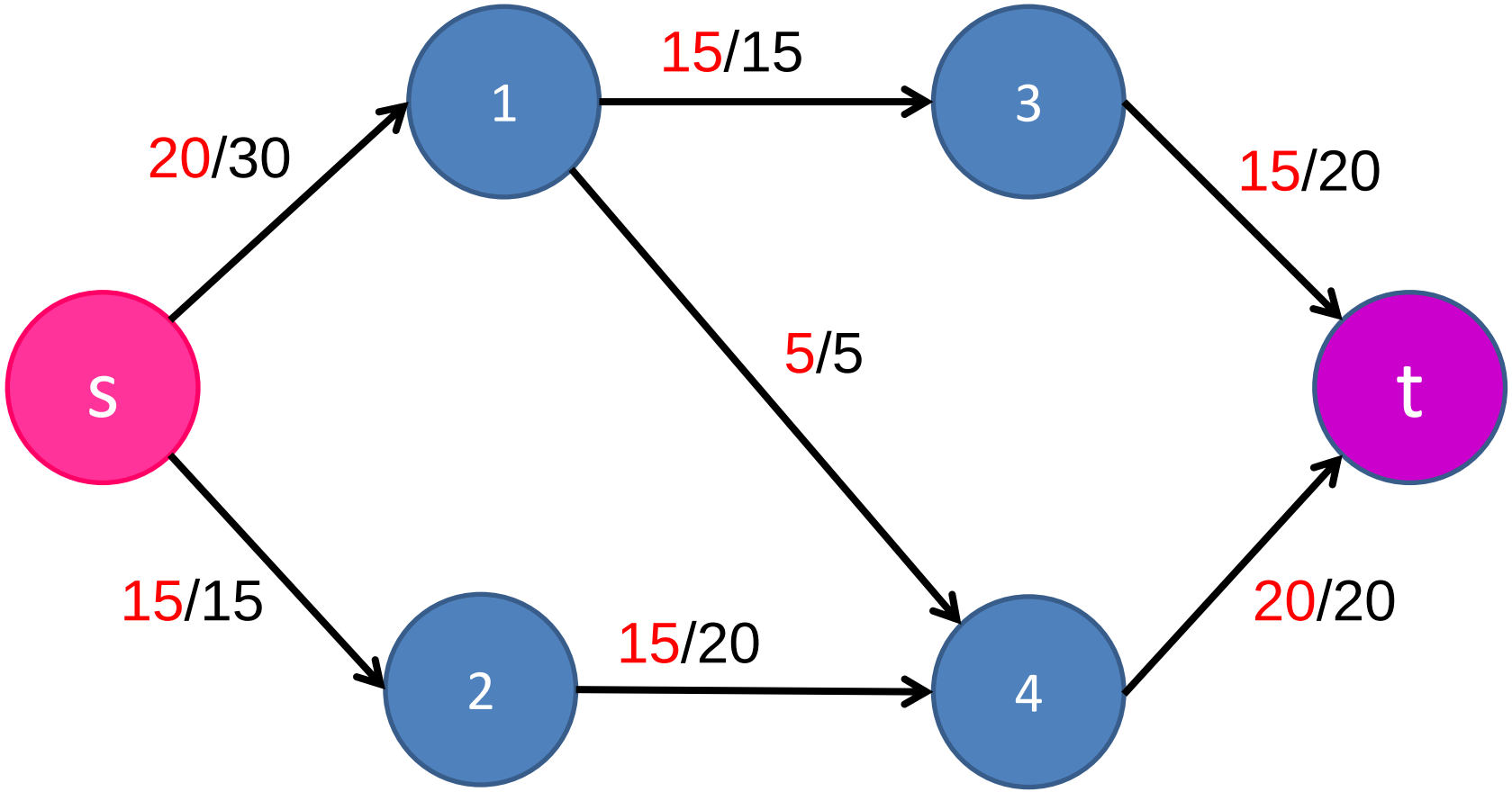
- Two vertice:
  - source **(S)**
  - sink **(T)**

enter from source and leave from sink.

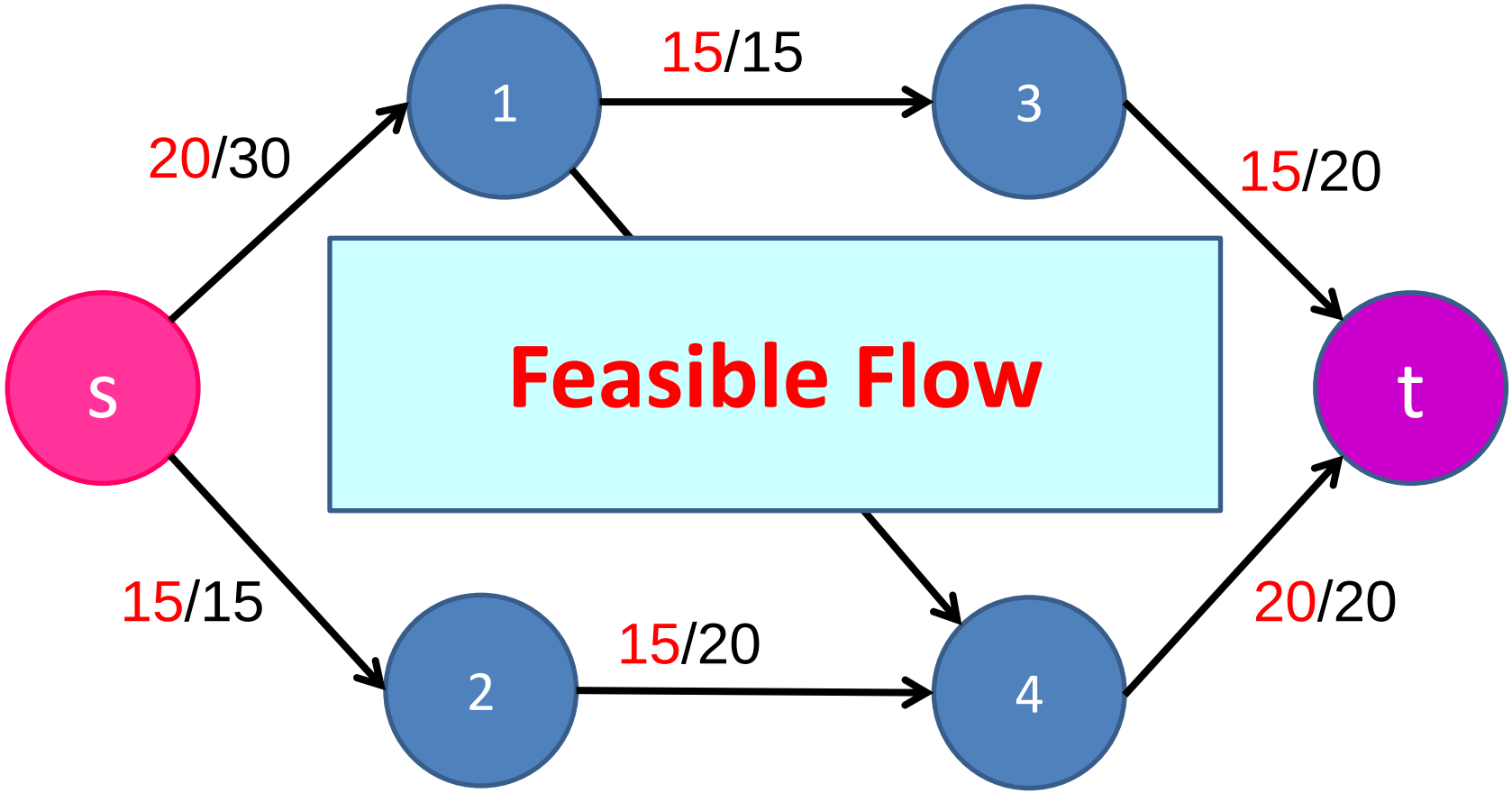
S-T Flow example



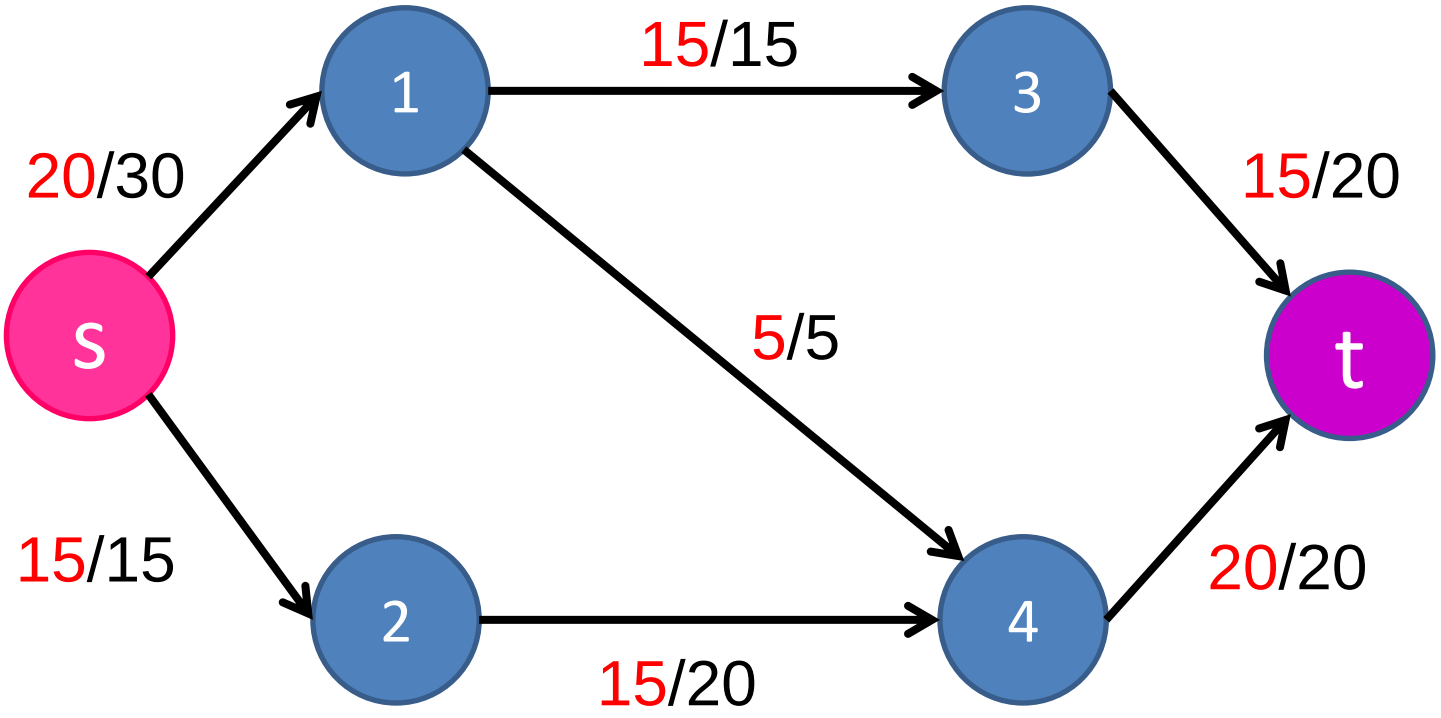
S-T Flow example



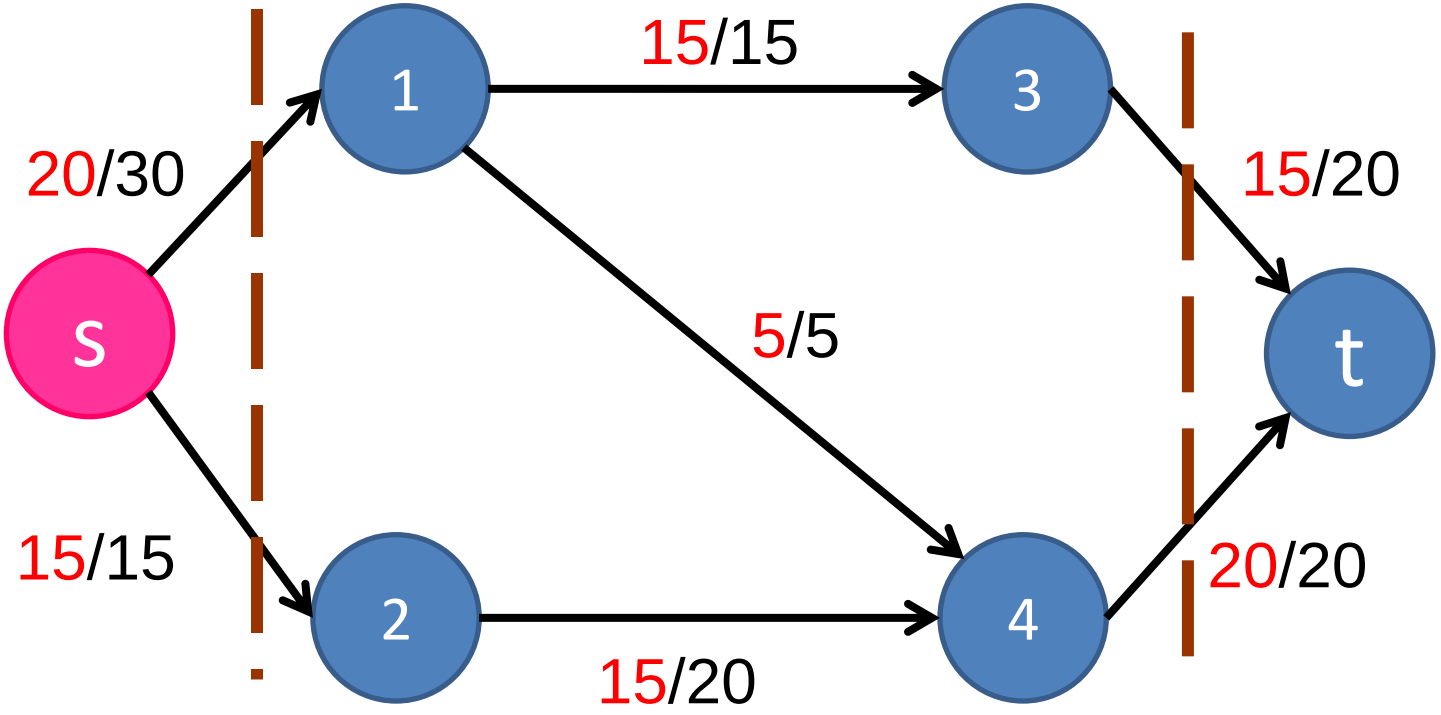
S-T Flow example



Flow conservation

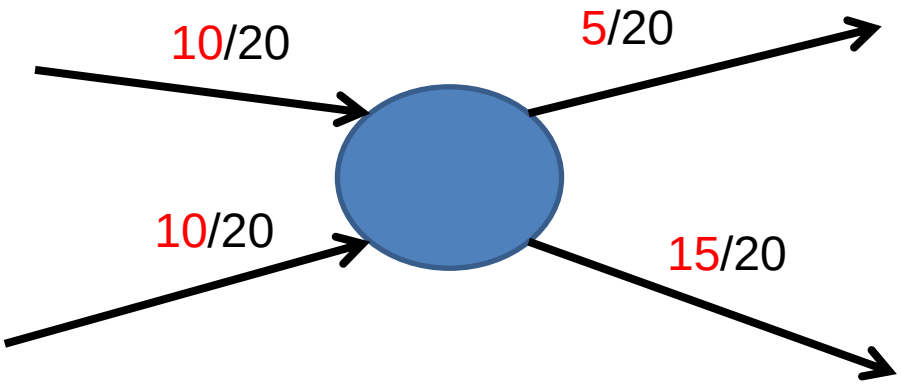


Flow conservation



Flow leave from source = Flow enter into sink

Flow conservation



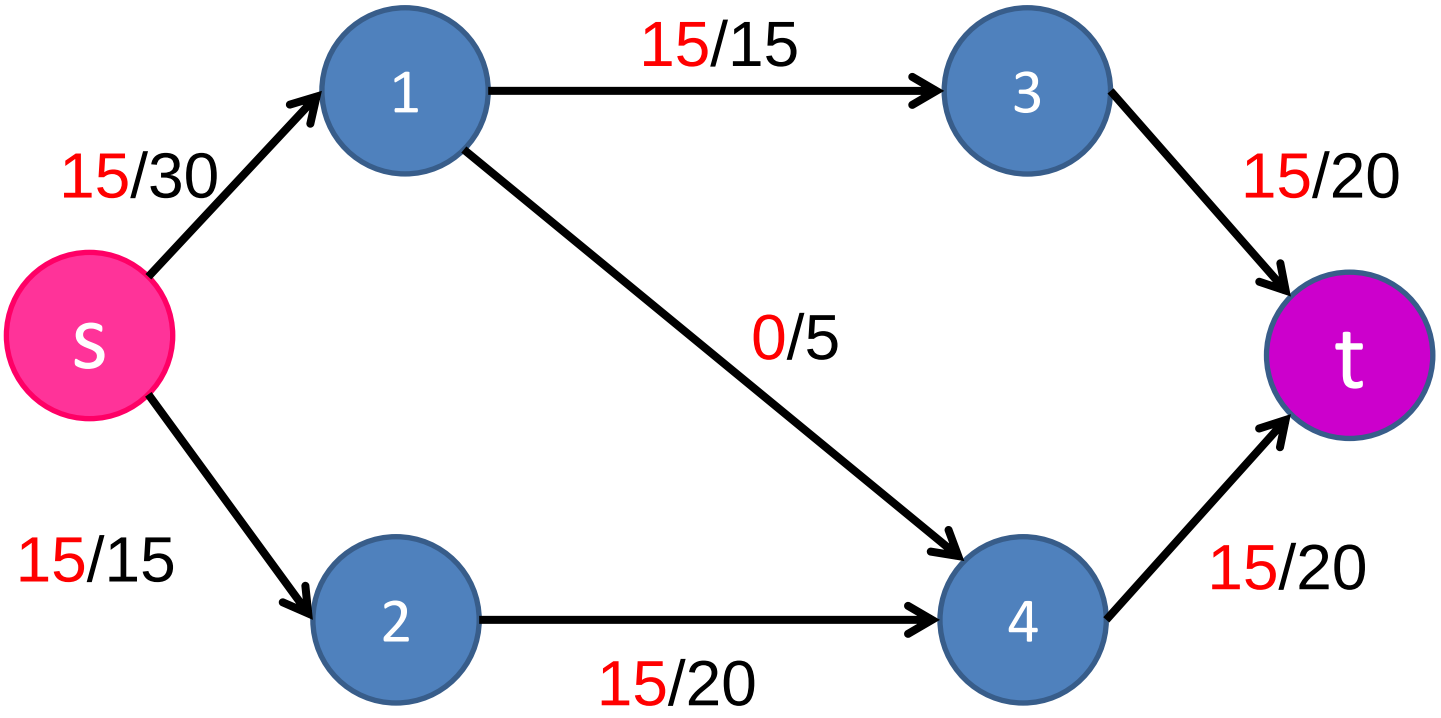
Flow enter into one node = Flow leave from one node

Residual Network

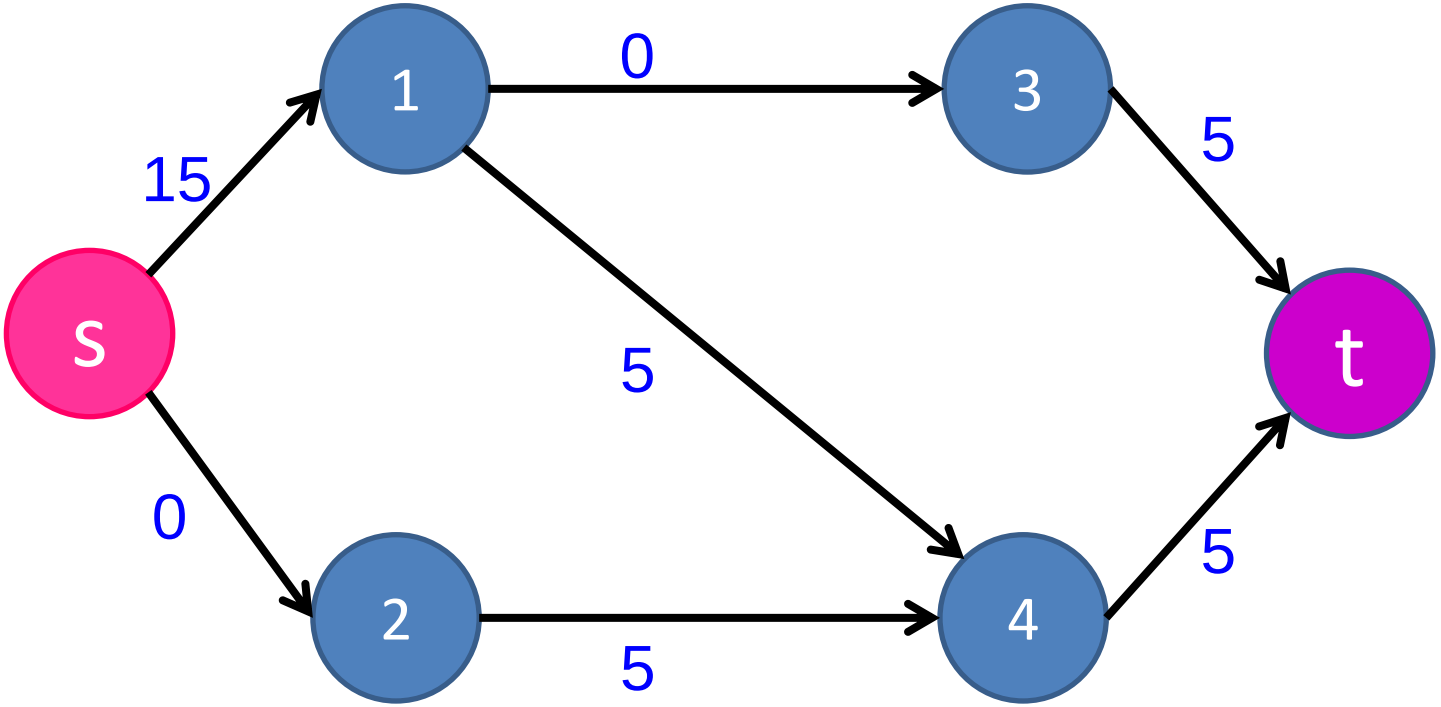
- **Definition:**

  a graph used to describe the remaining capacity of all edges.

Residual Network



Residual Network

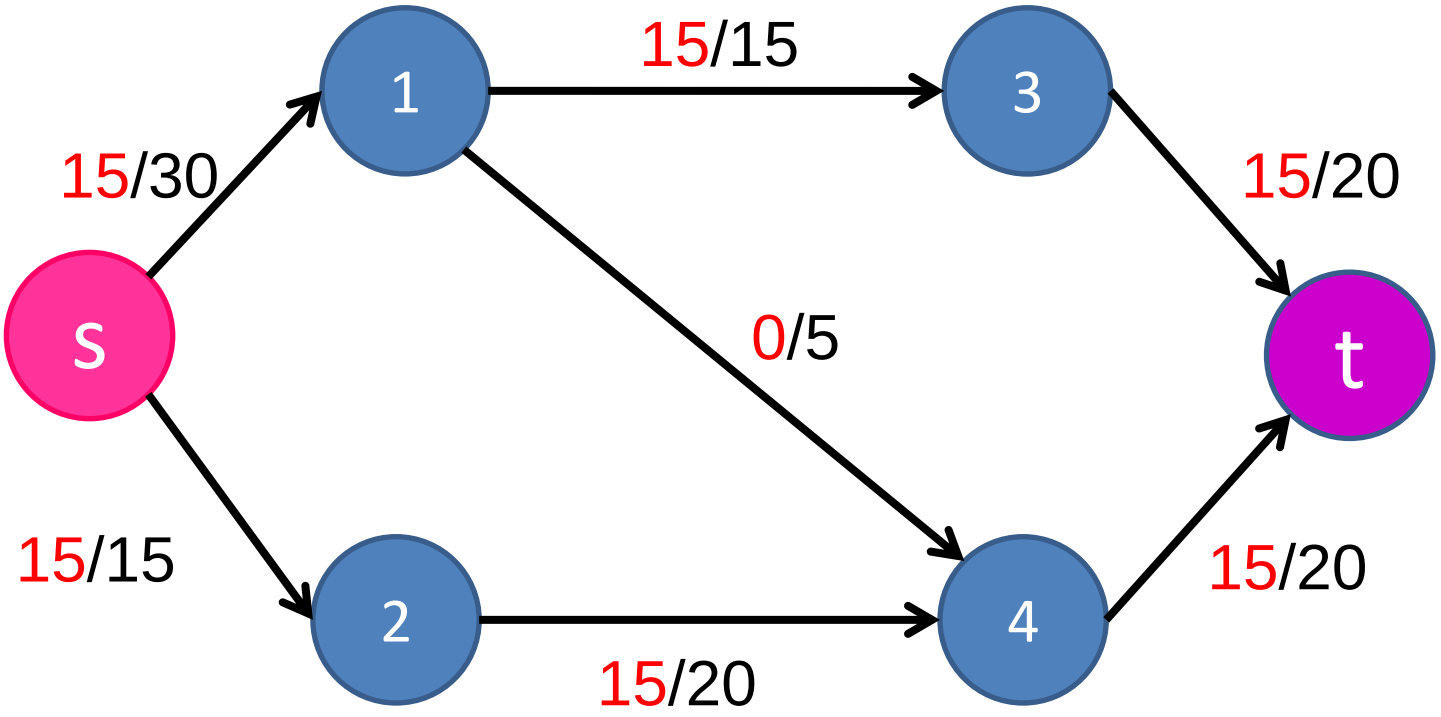


Augmenting Path

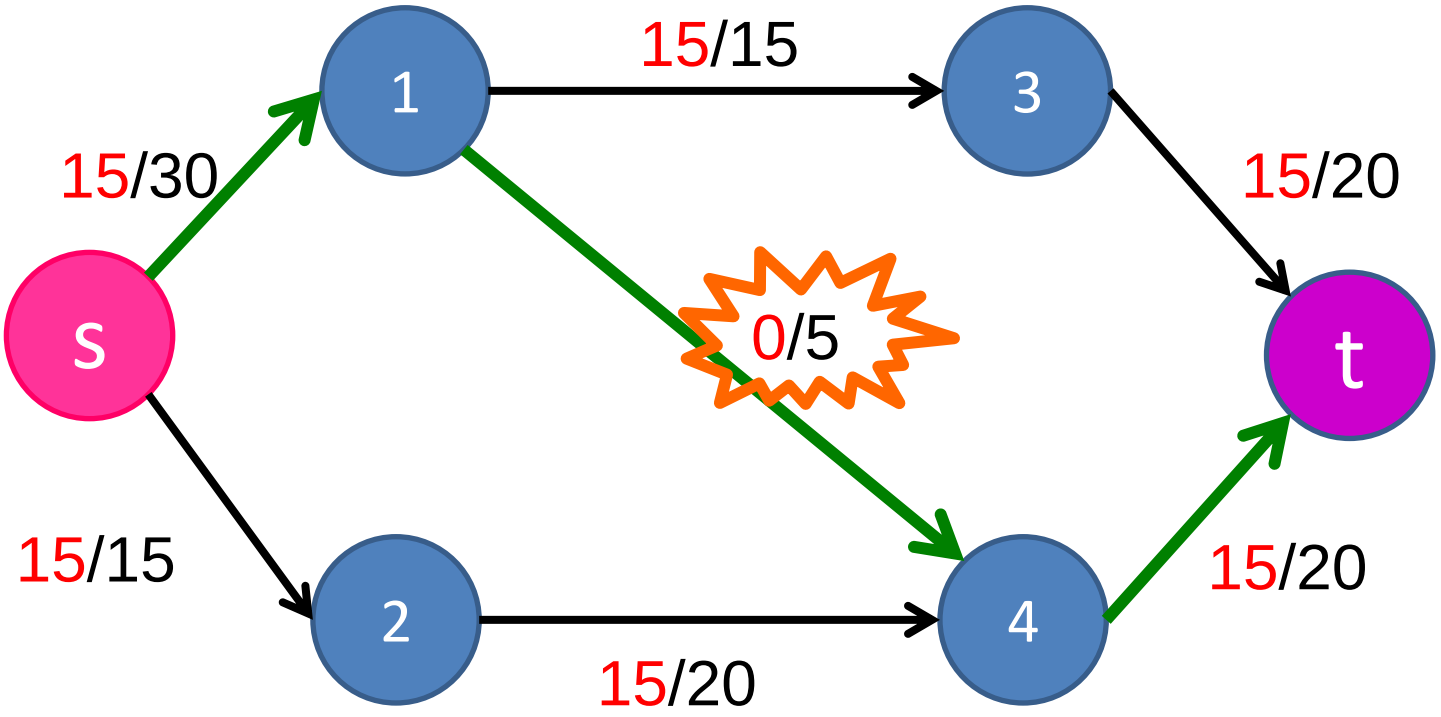
- **Definition:**

  A path form source to sink in residual network that all capacity of edges are greater than zero.

Augmenting Path

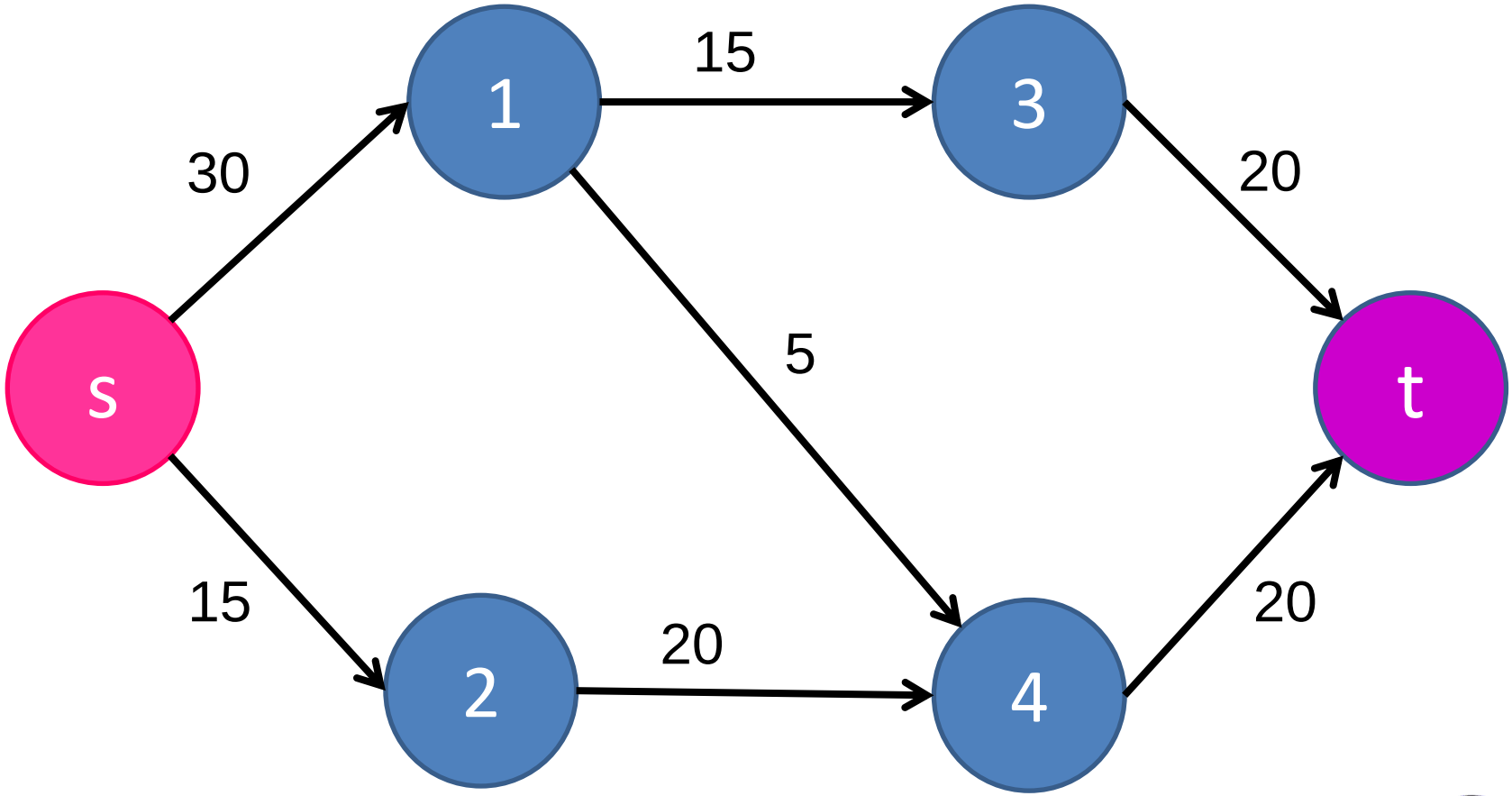


Augmenting Path

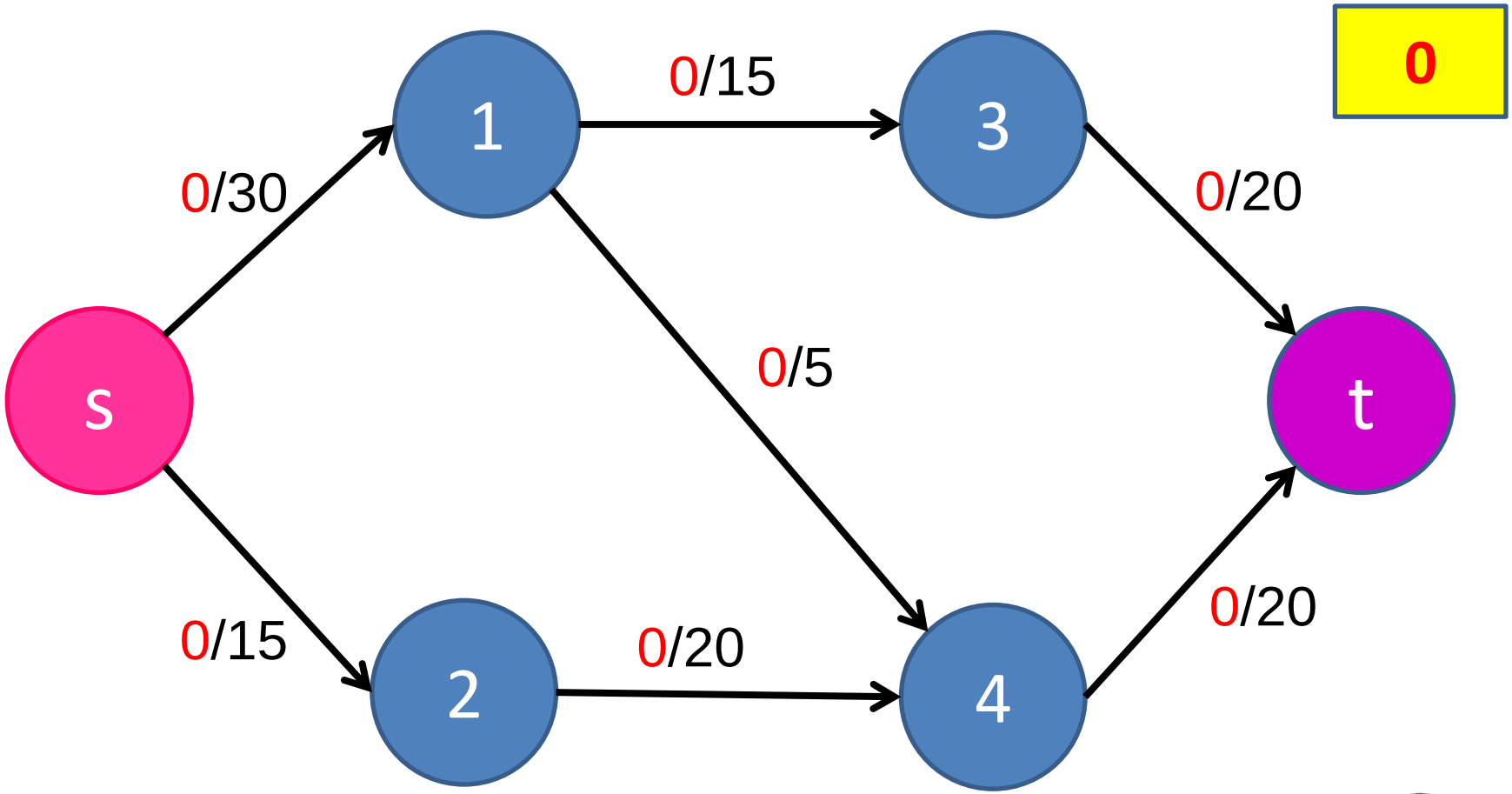


Still capable of a flow with 5

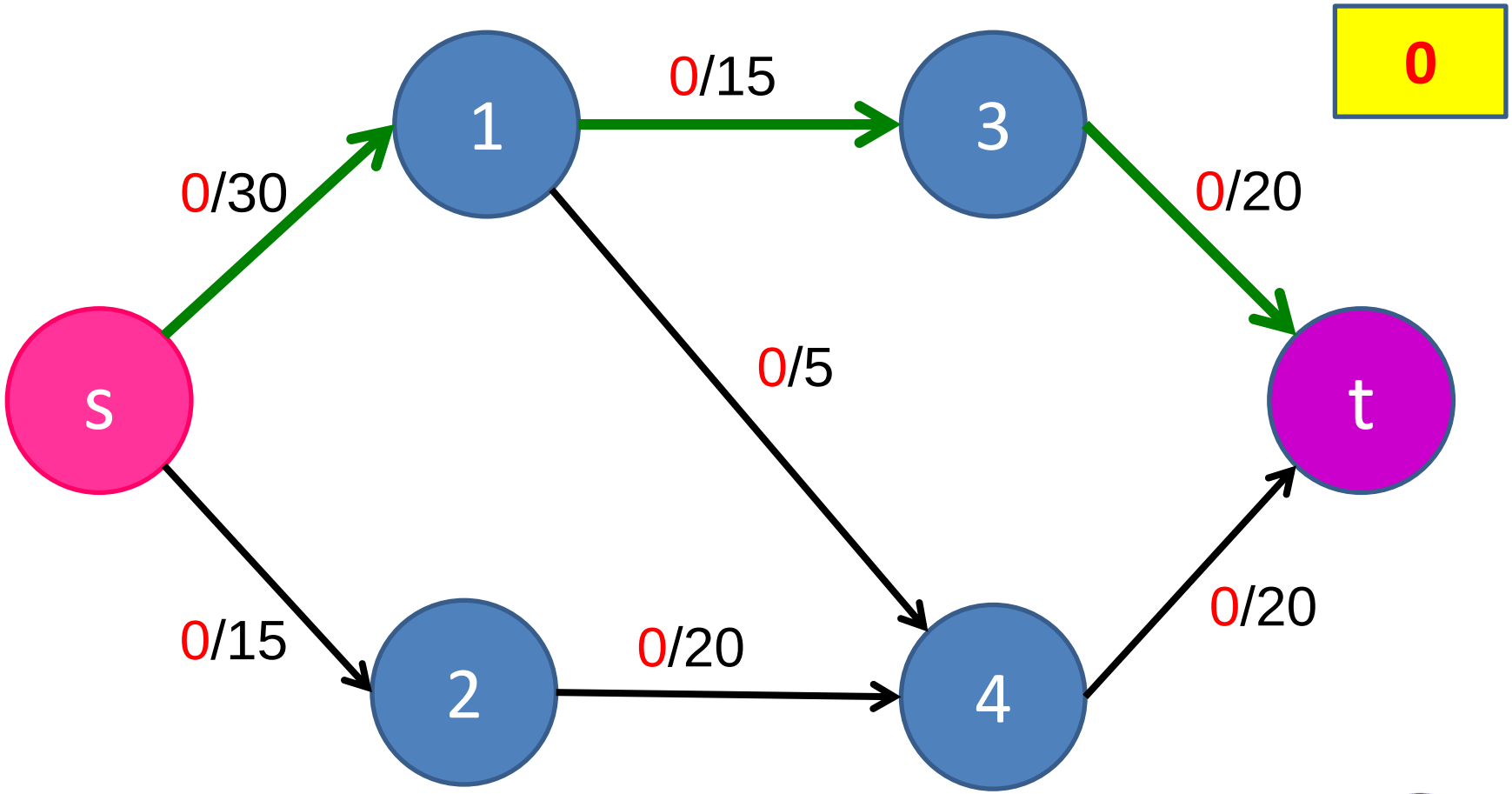
Maximum Flow



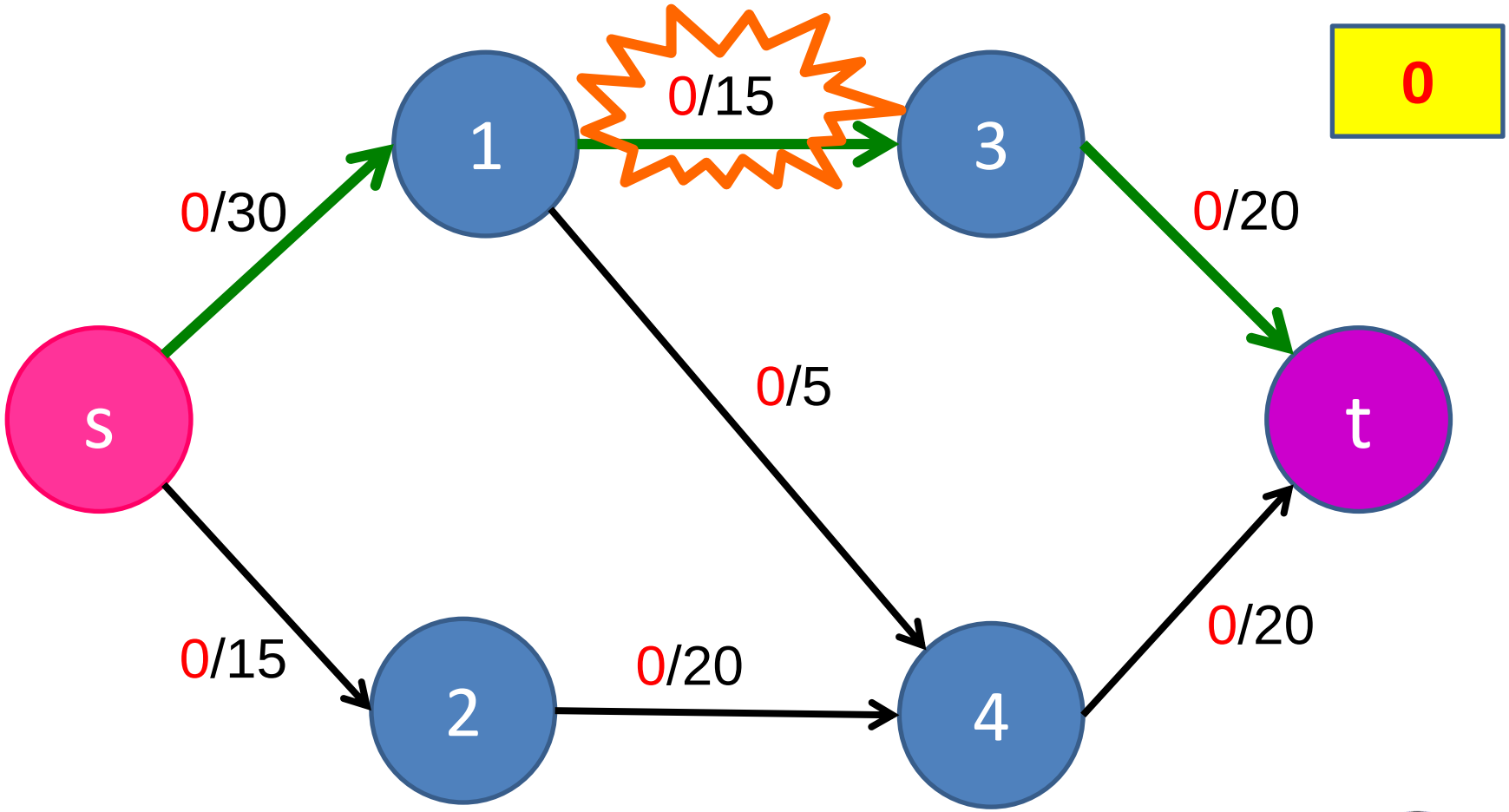
Maximum Flow



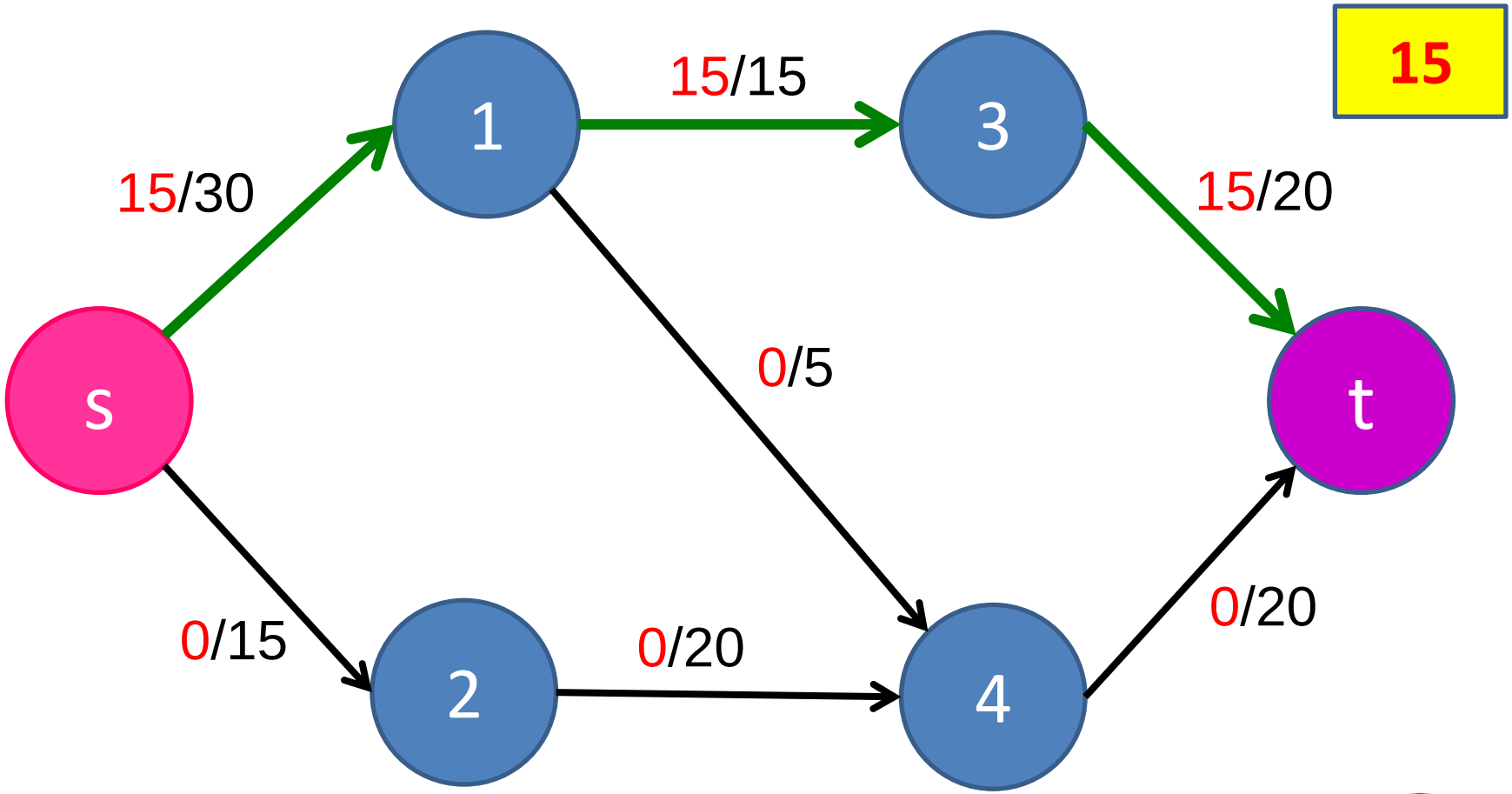
Find augmenting path



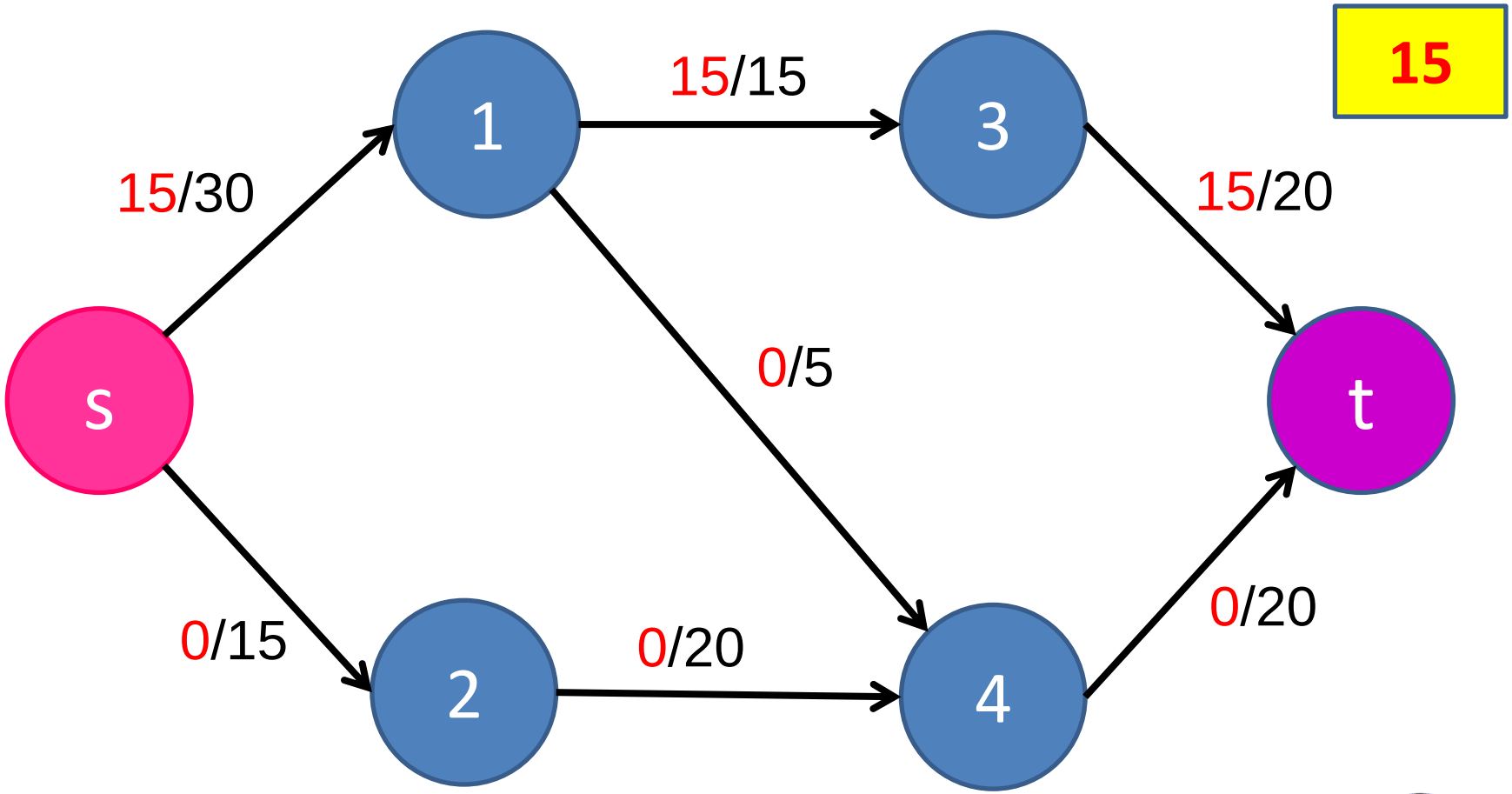
Find bottleneck



Flow pass



Find augmenting path



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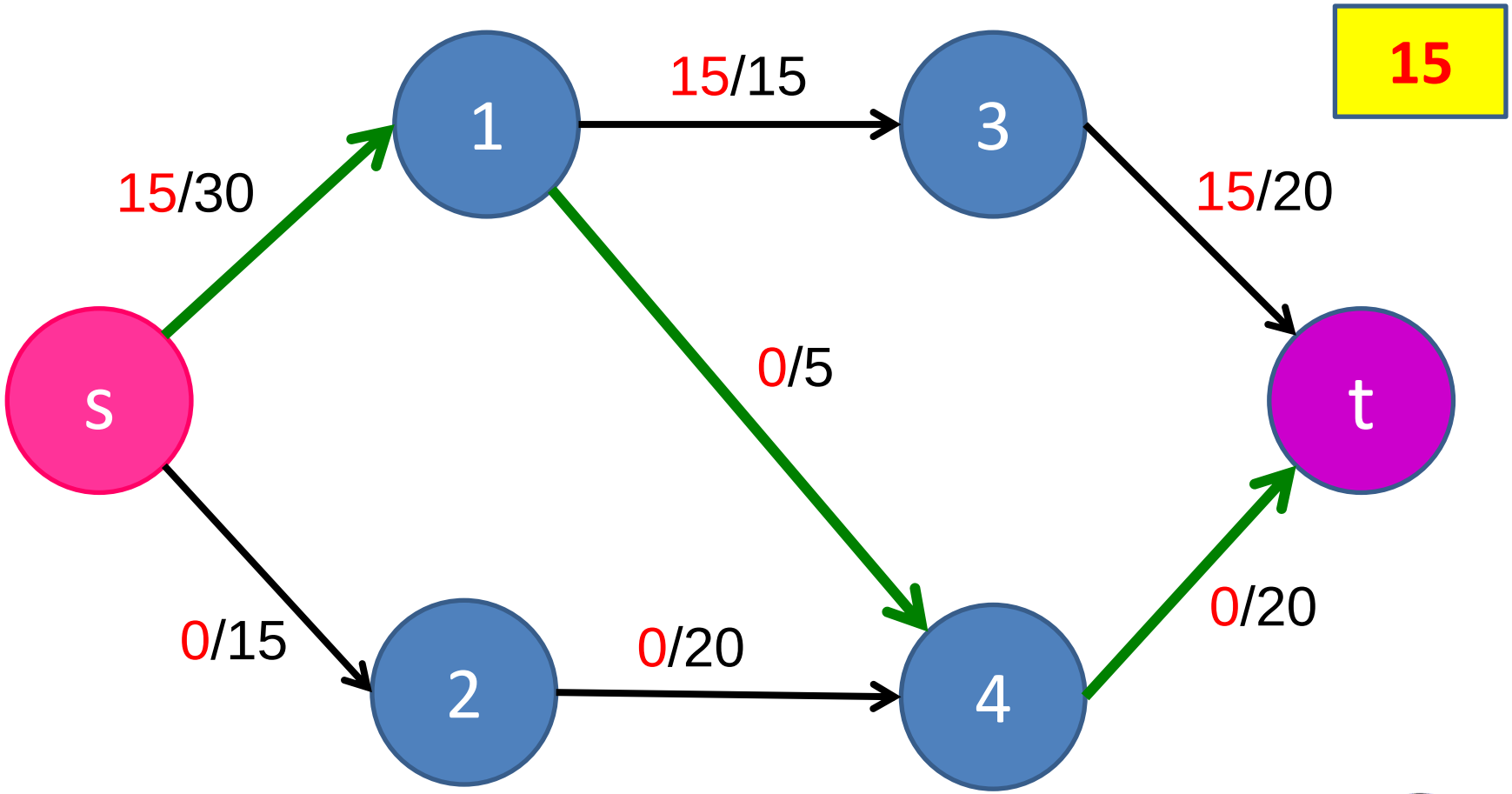
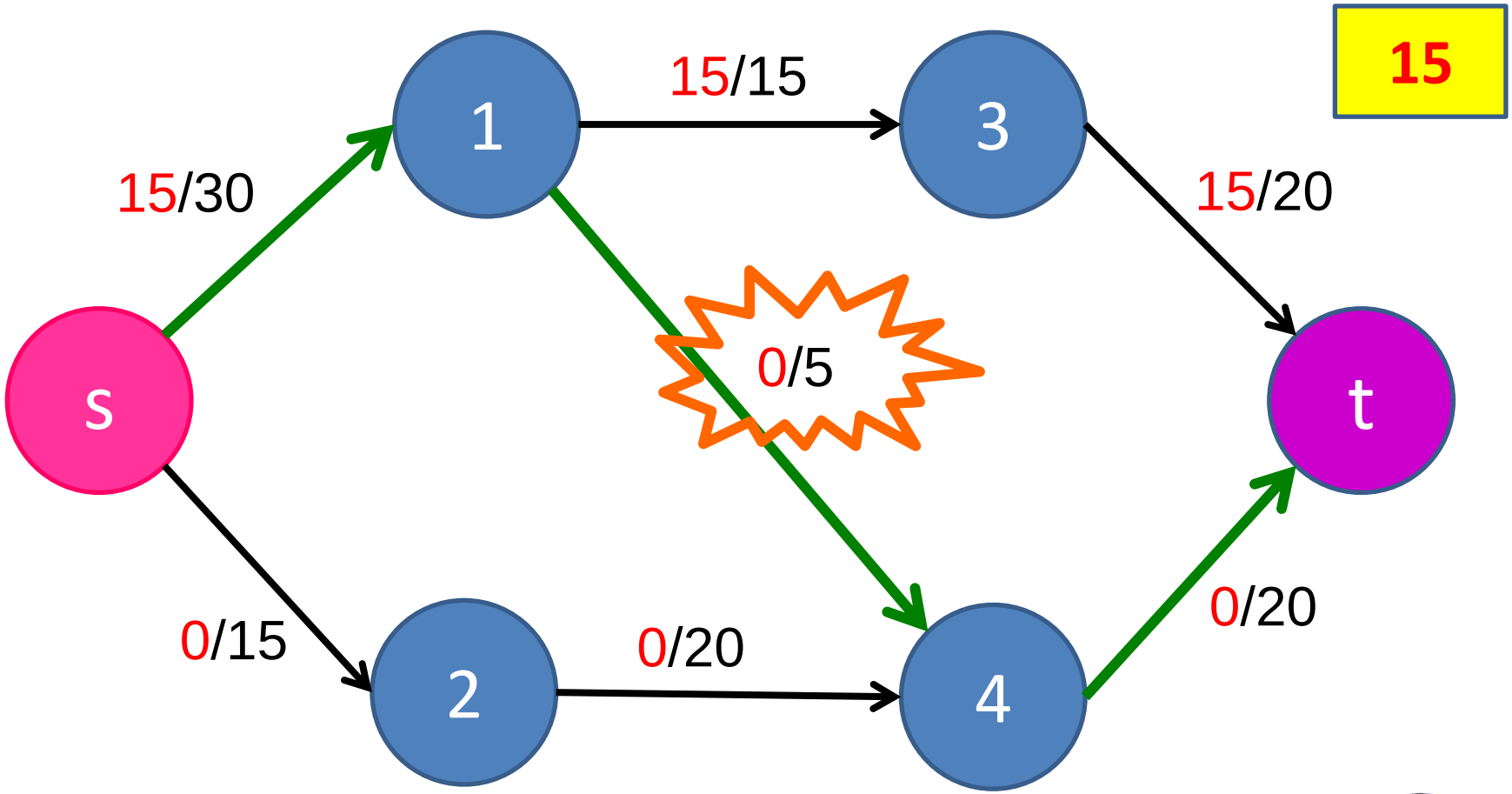


Find bottleneck



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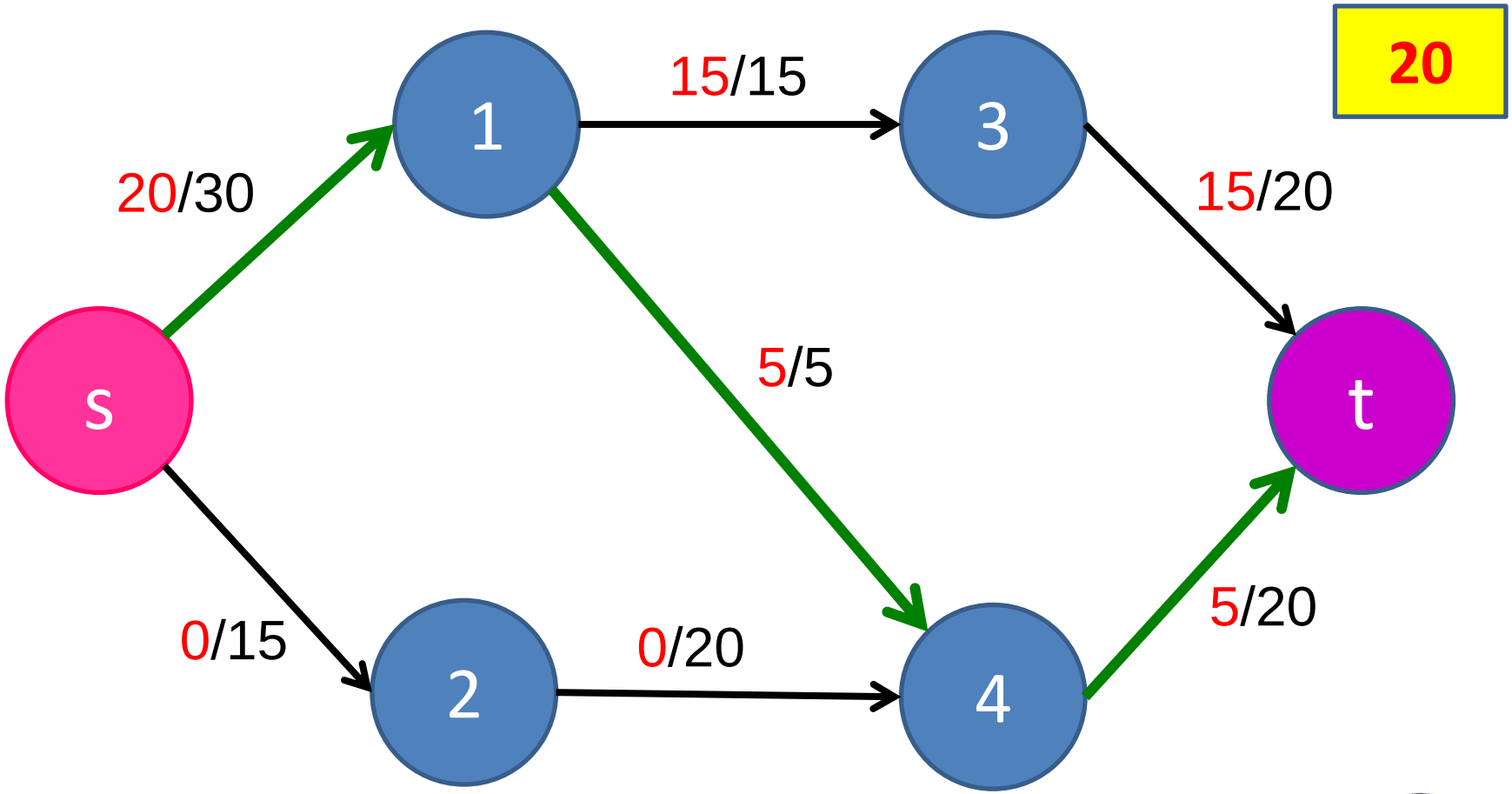

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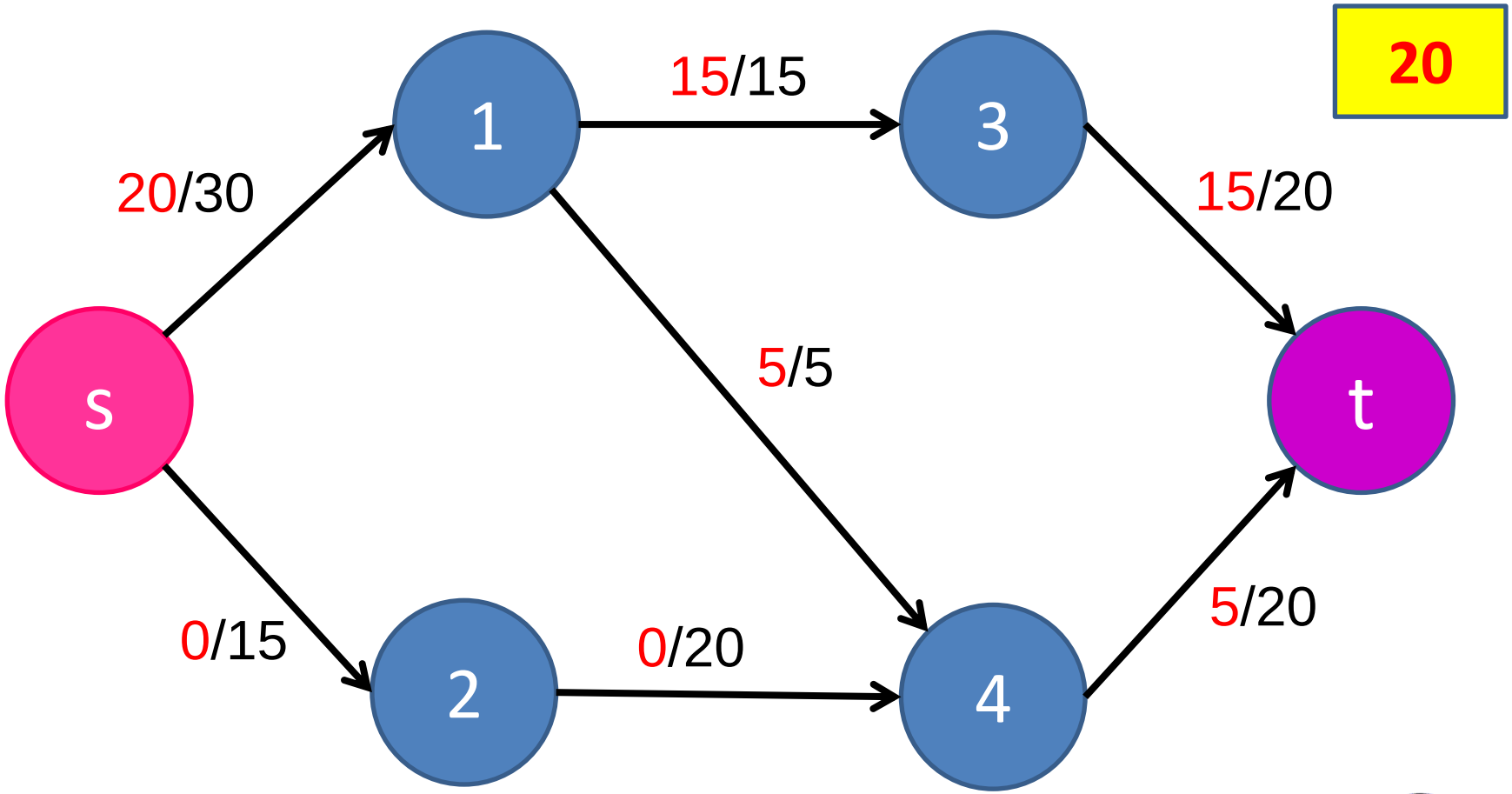

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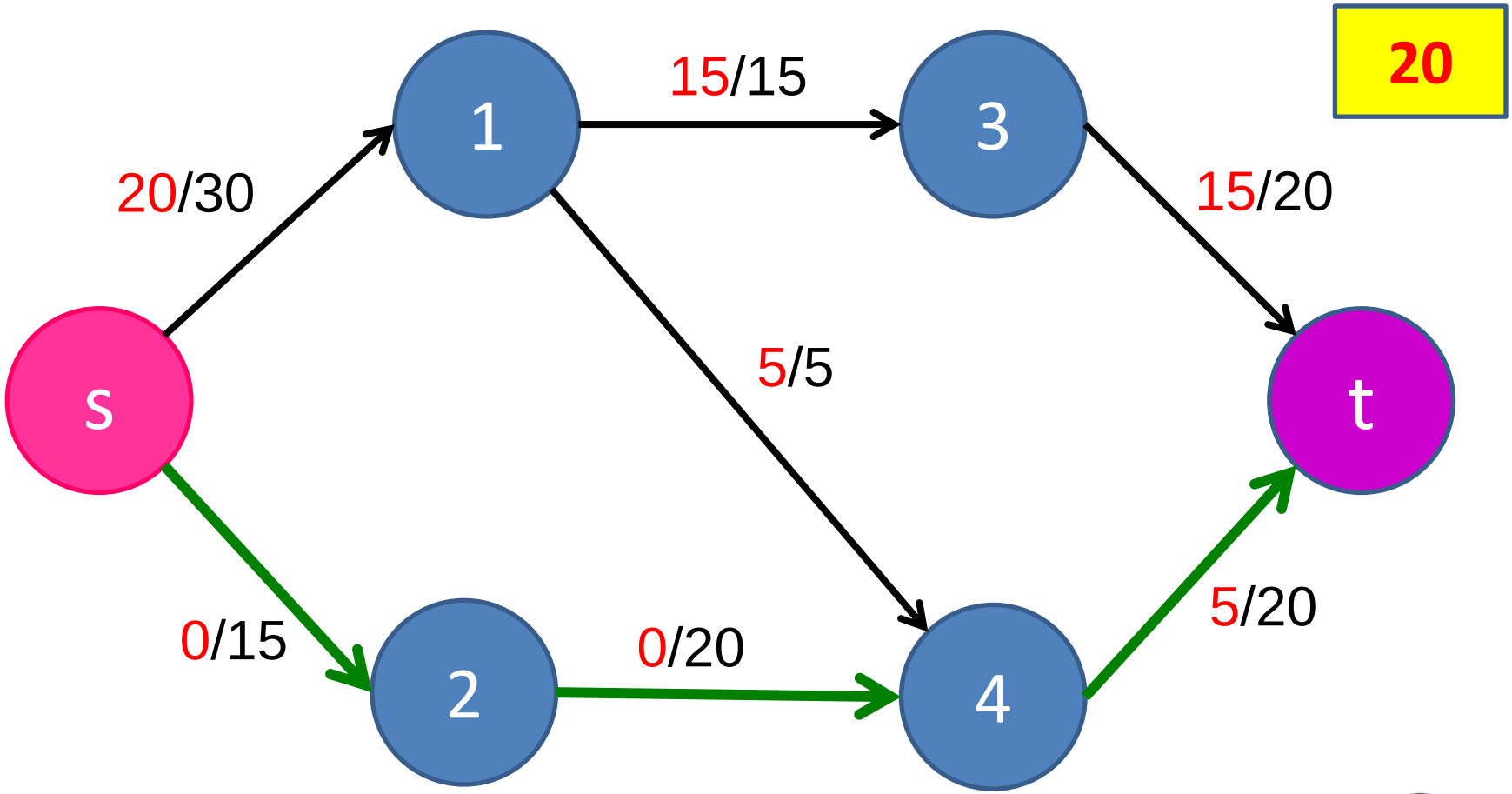

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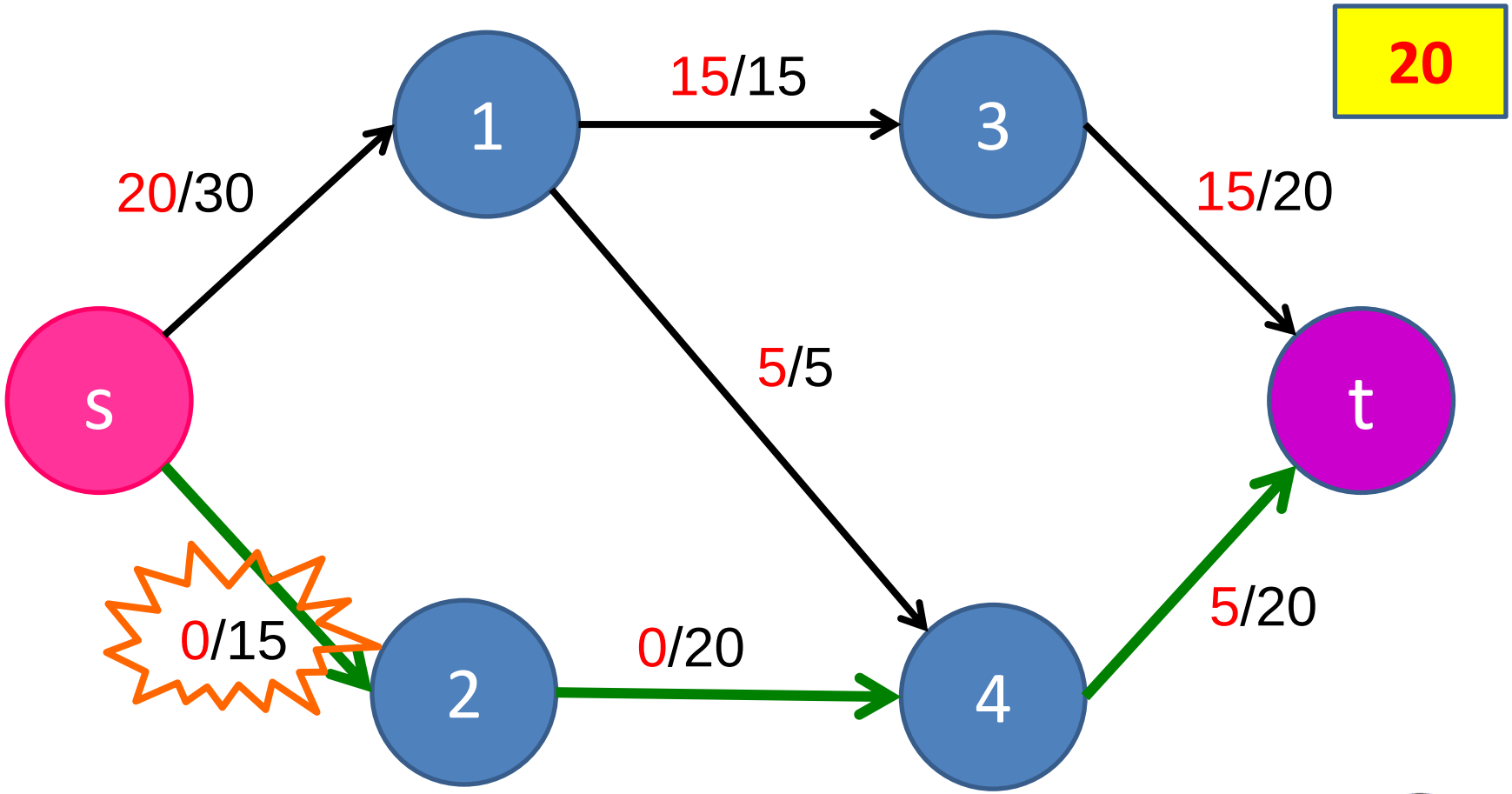

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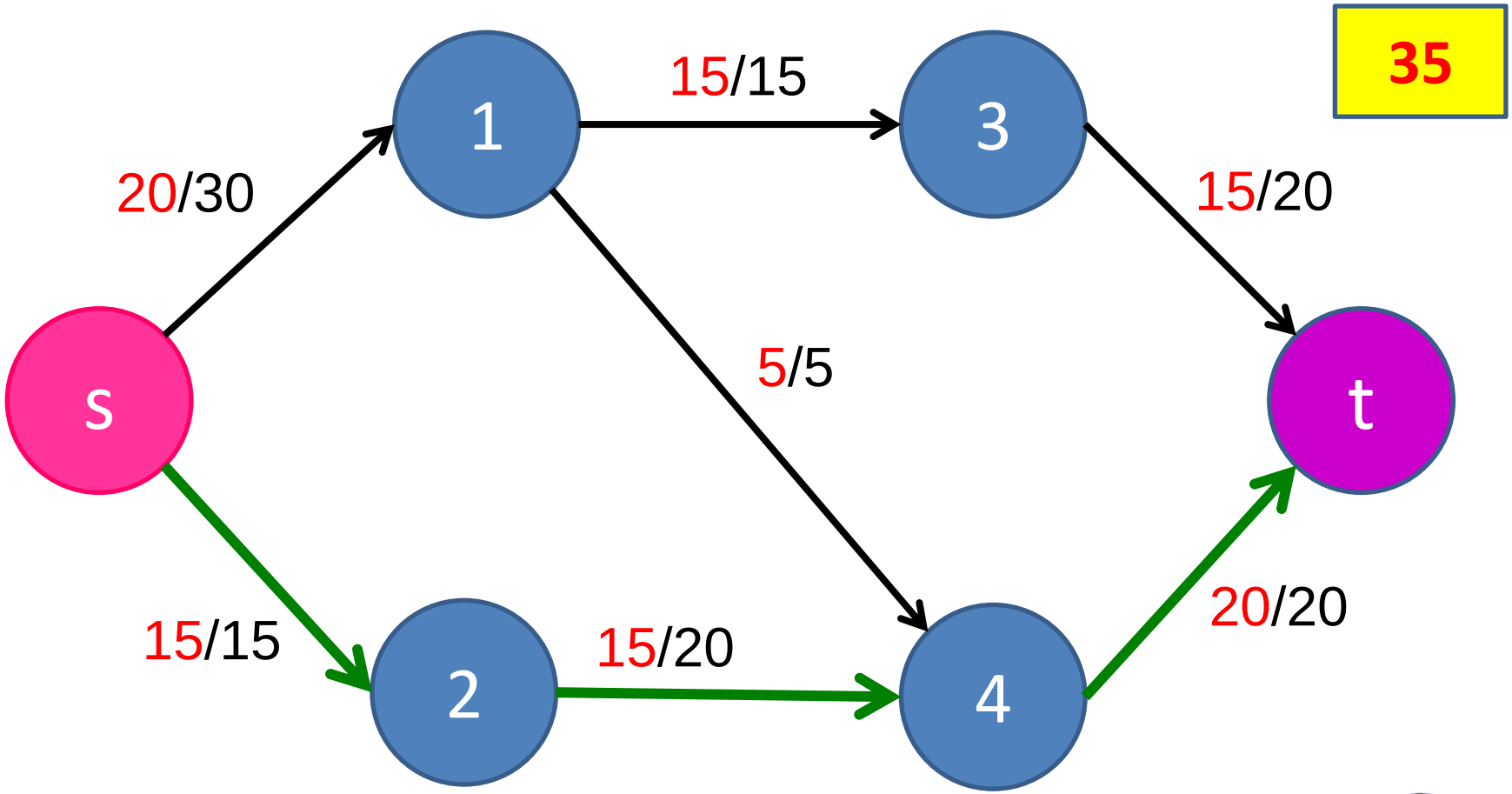

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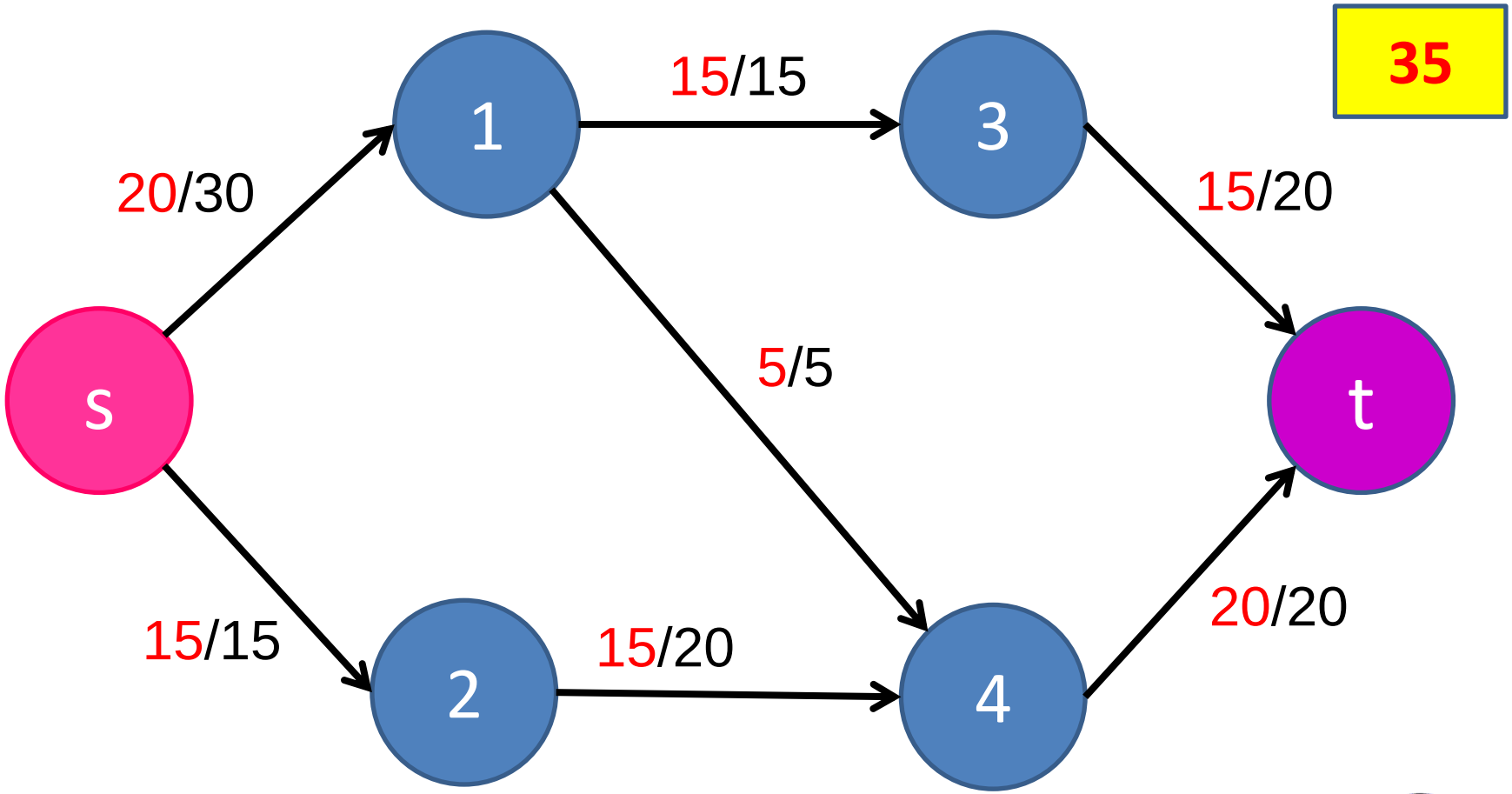

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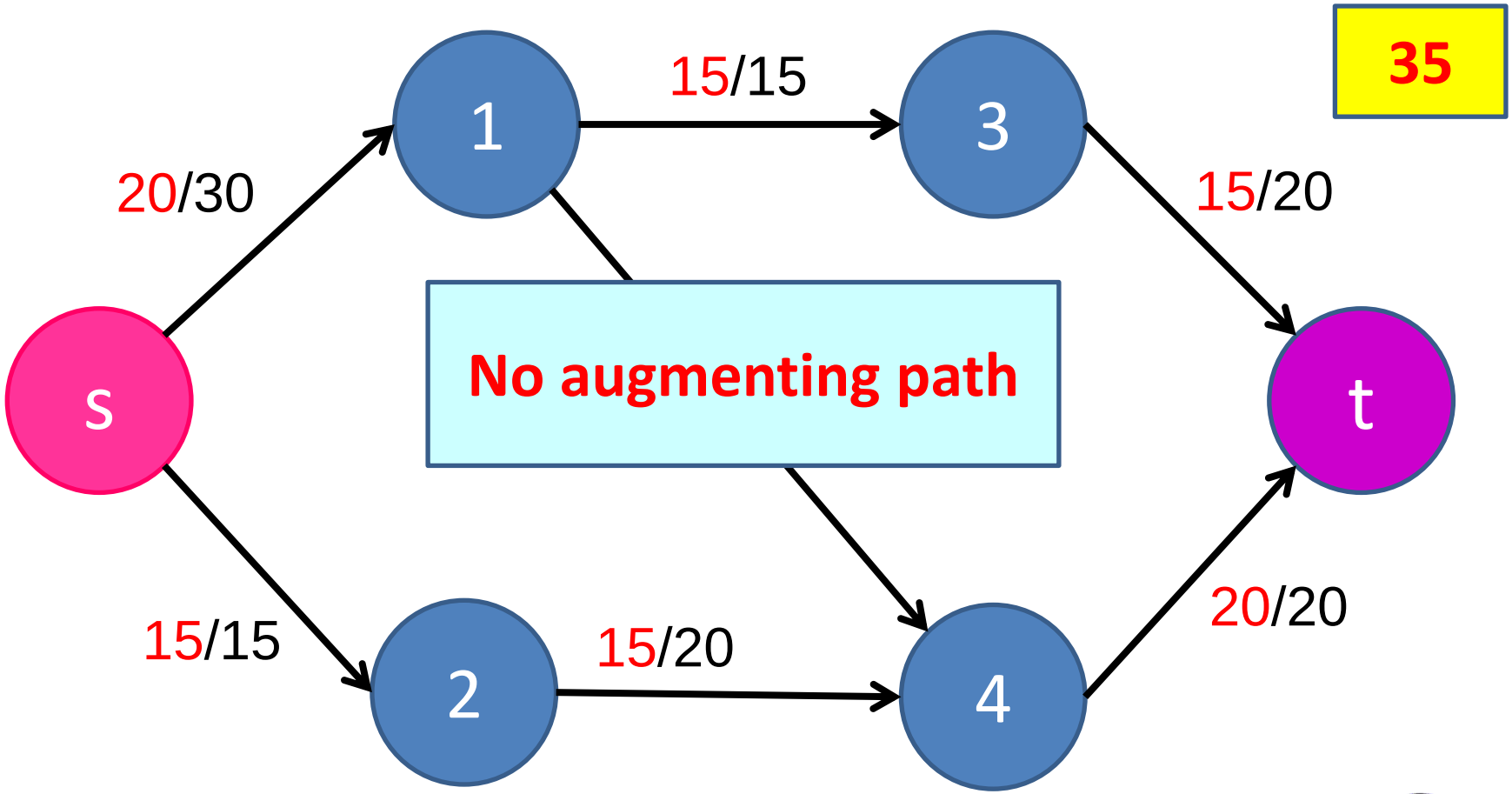
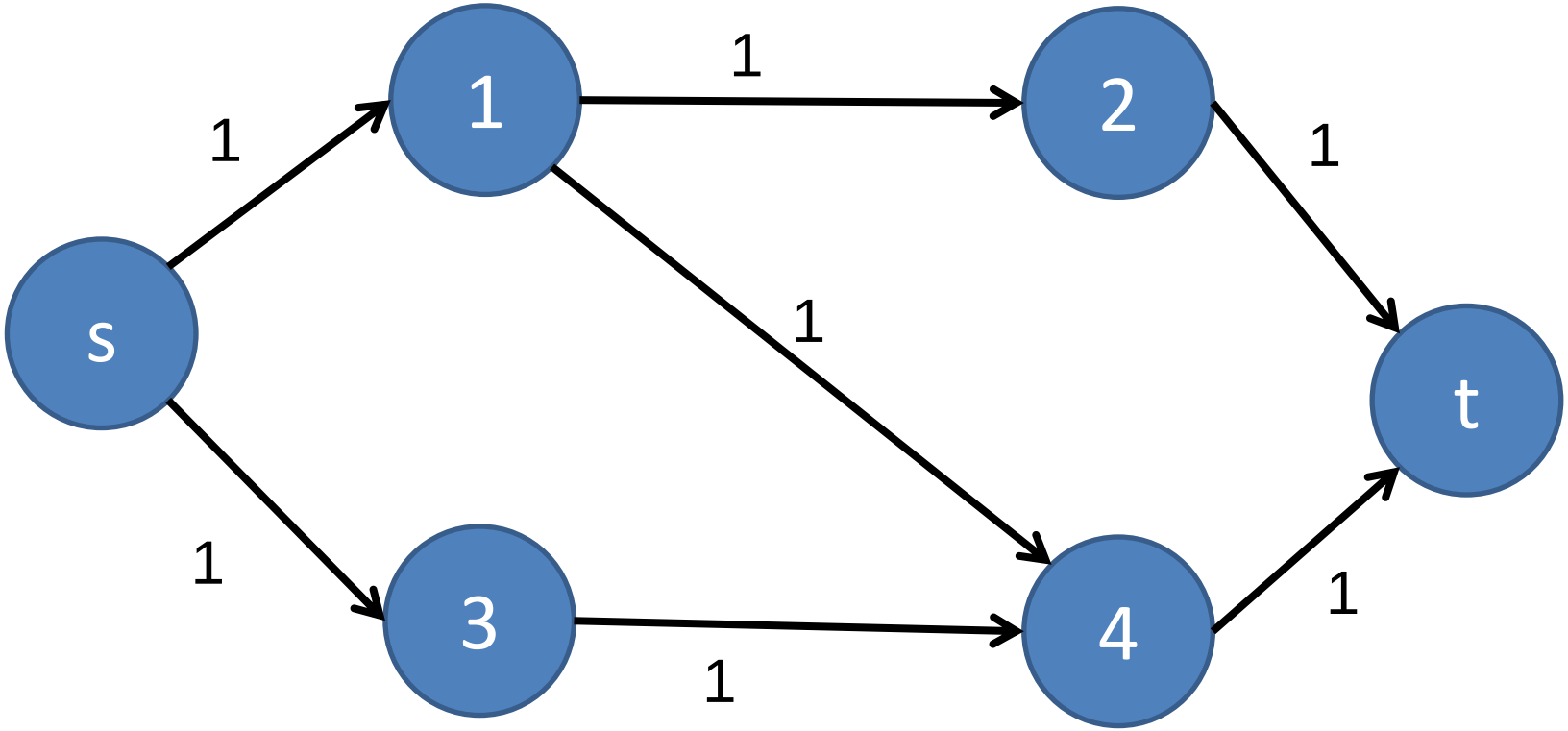


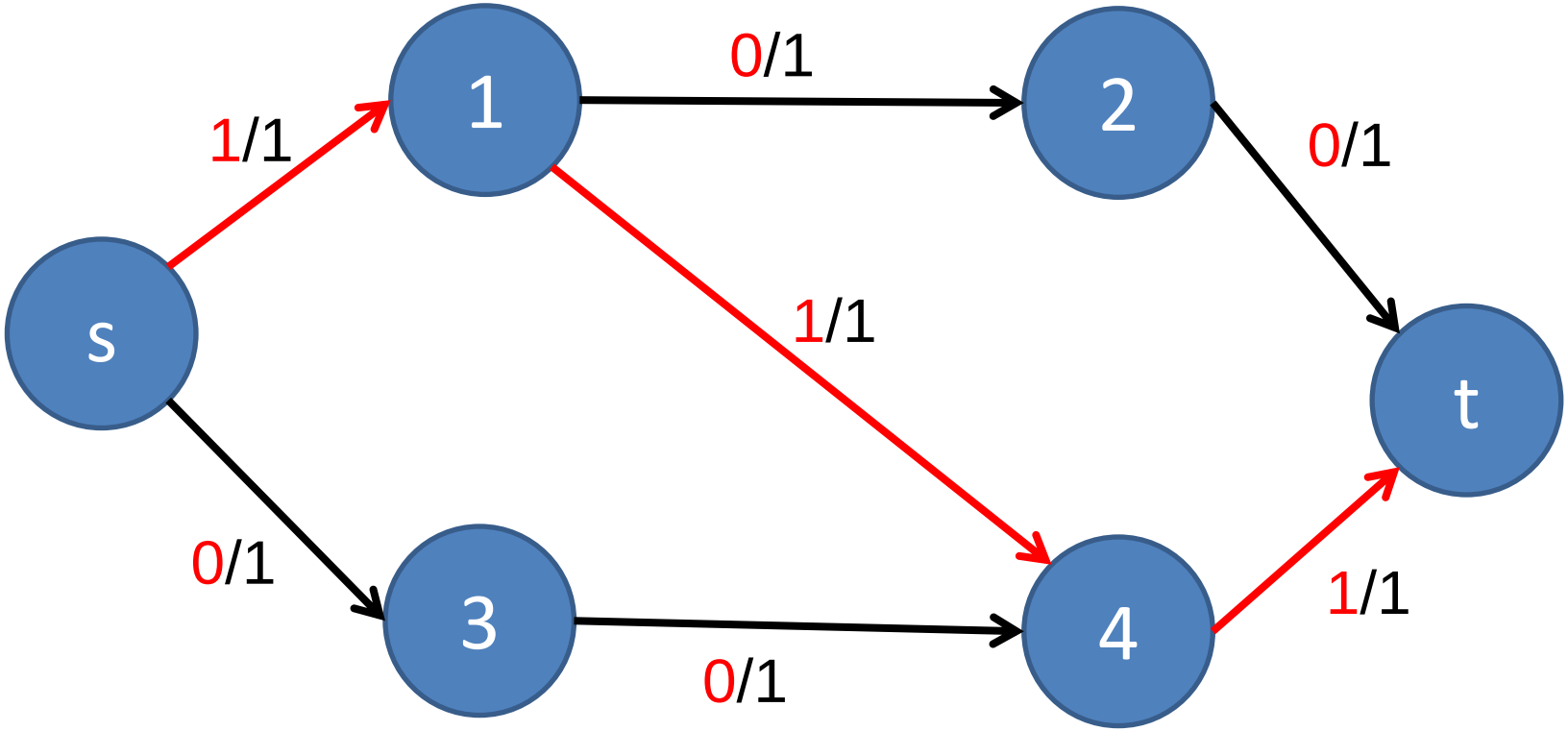
Maximum Flow

Just DFS ?

Opposite edge

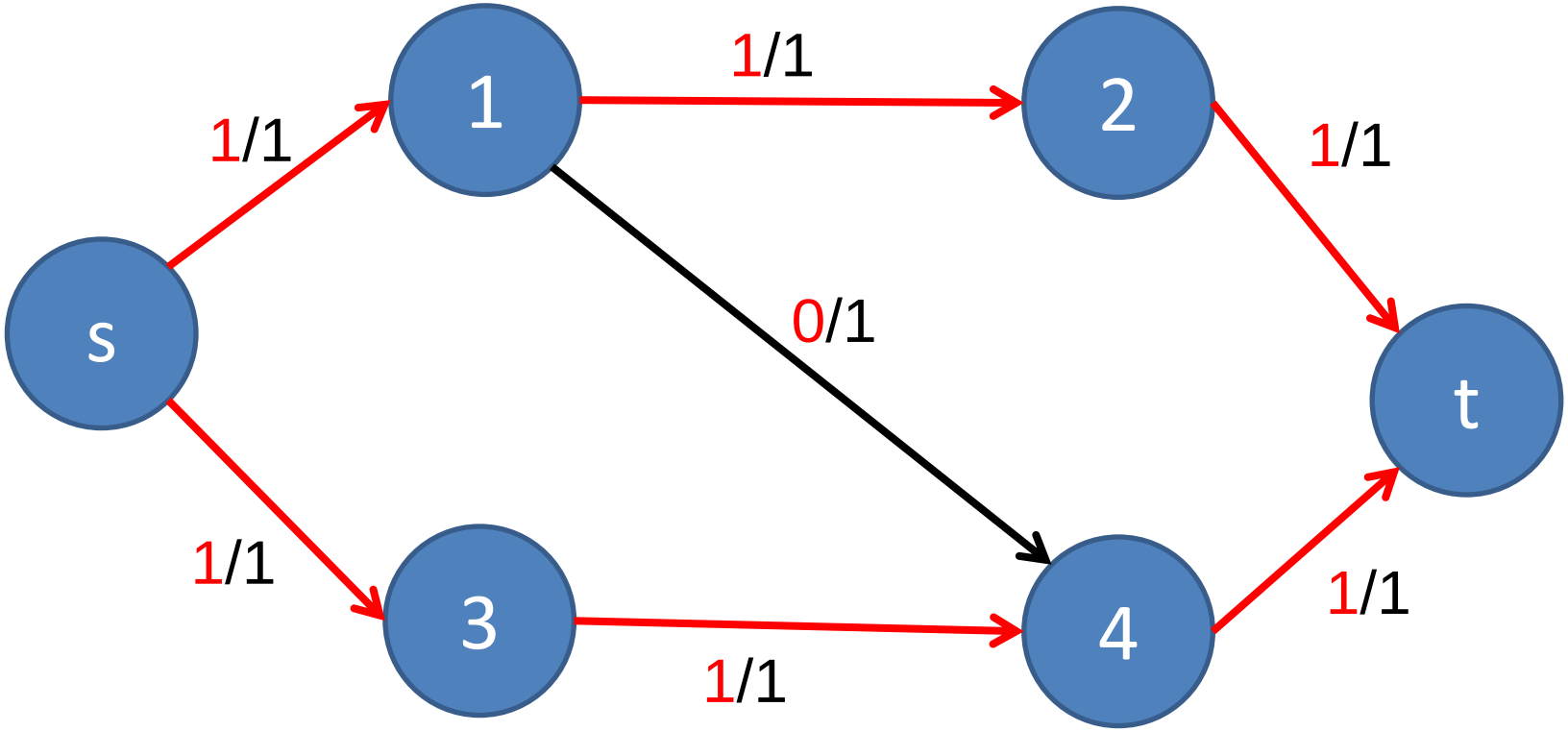


Opposite edge - 1



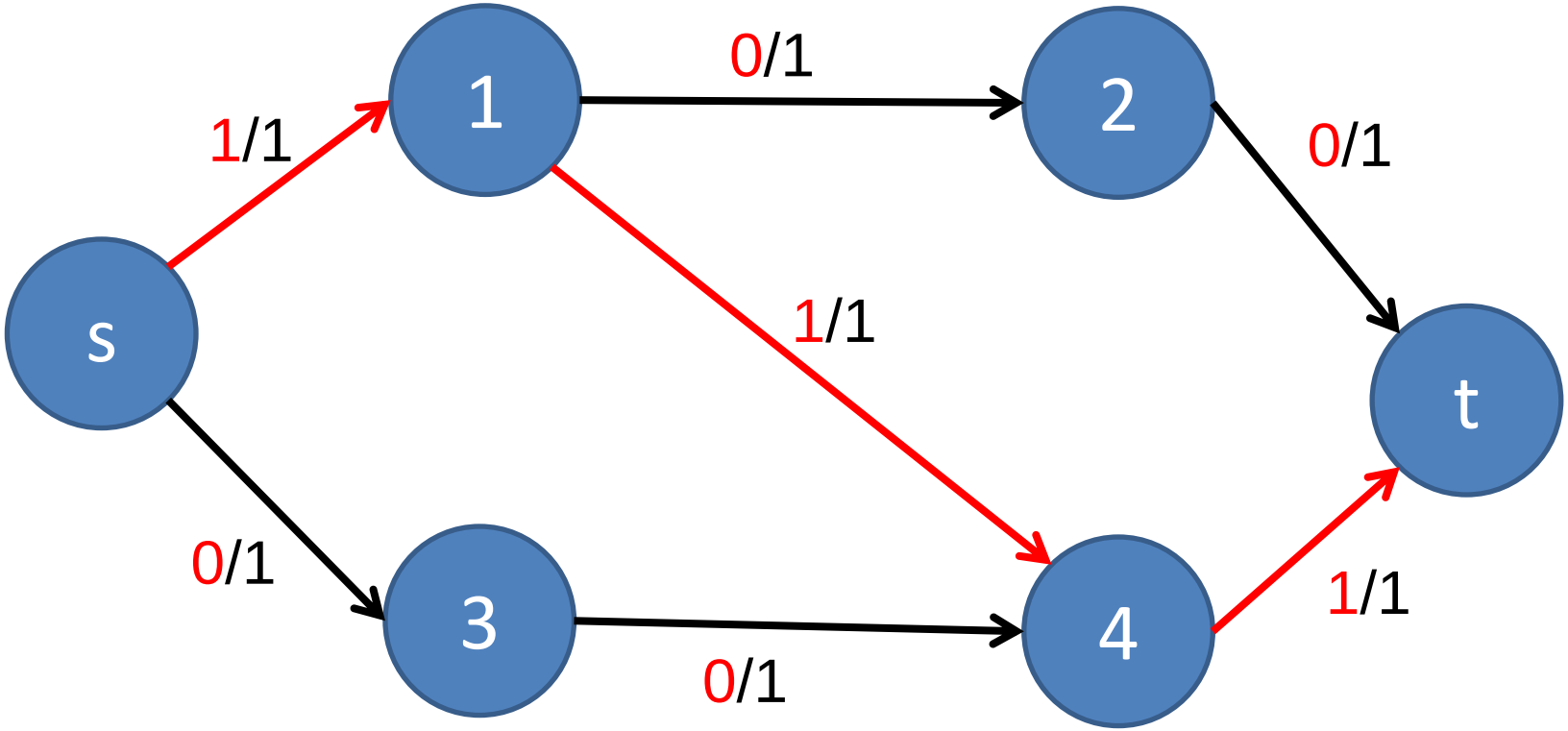
Maximum flow = 1 ????

Opposite edge - 1



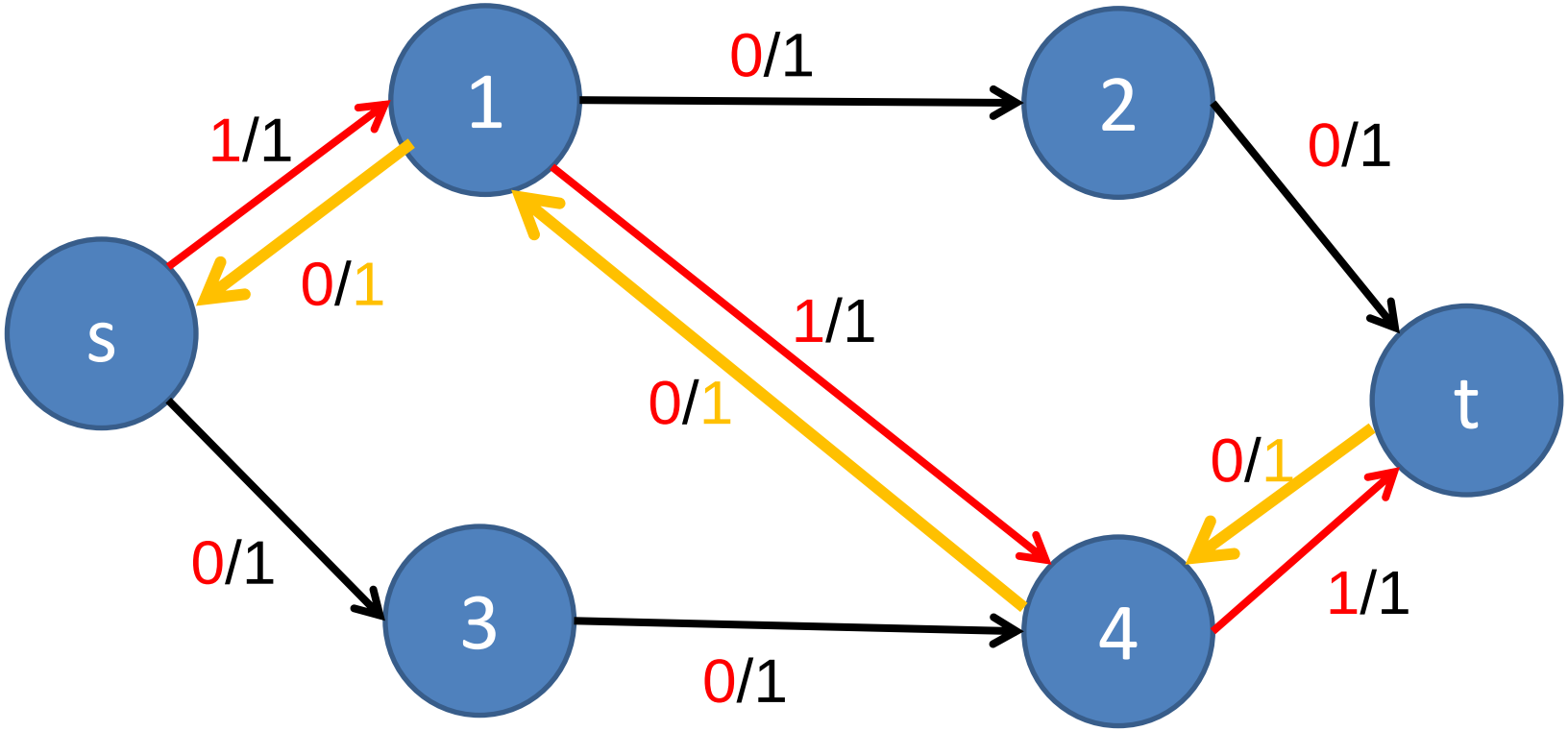
Maximum flow = 2 !!!

Opposite edge - 2

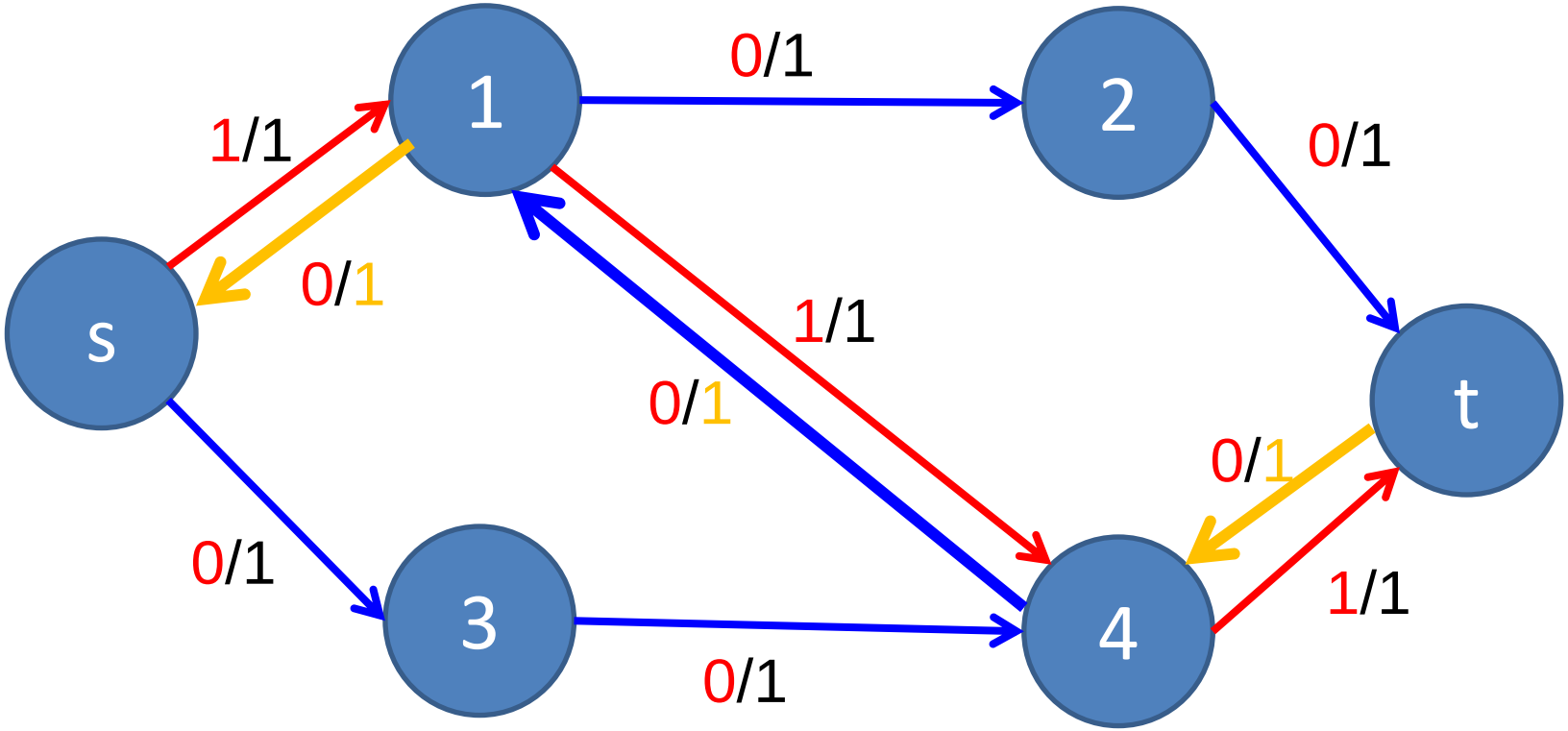


Maximum flow = 1 ????

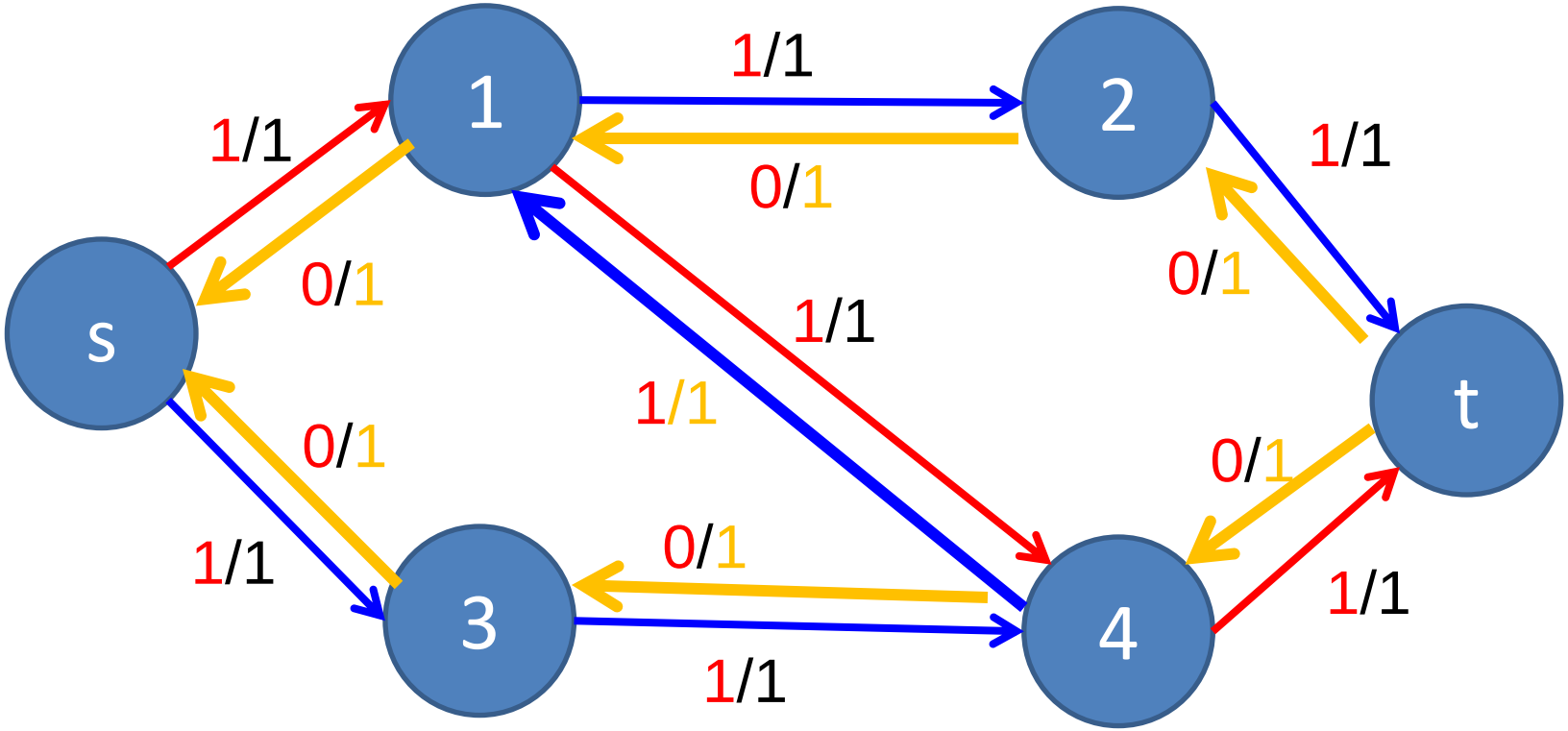
Opposite edge - 2



Opposite edge - 2

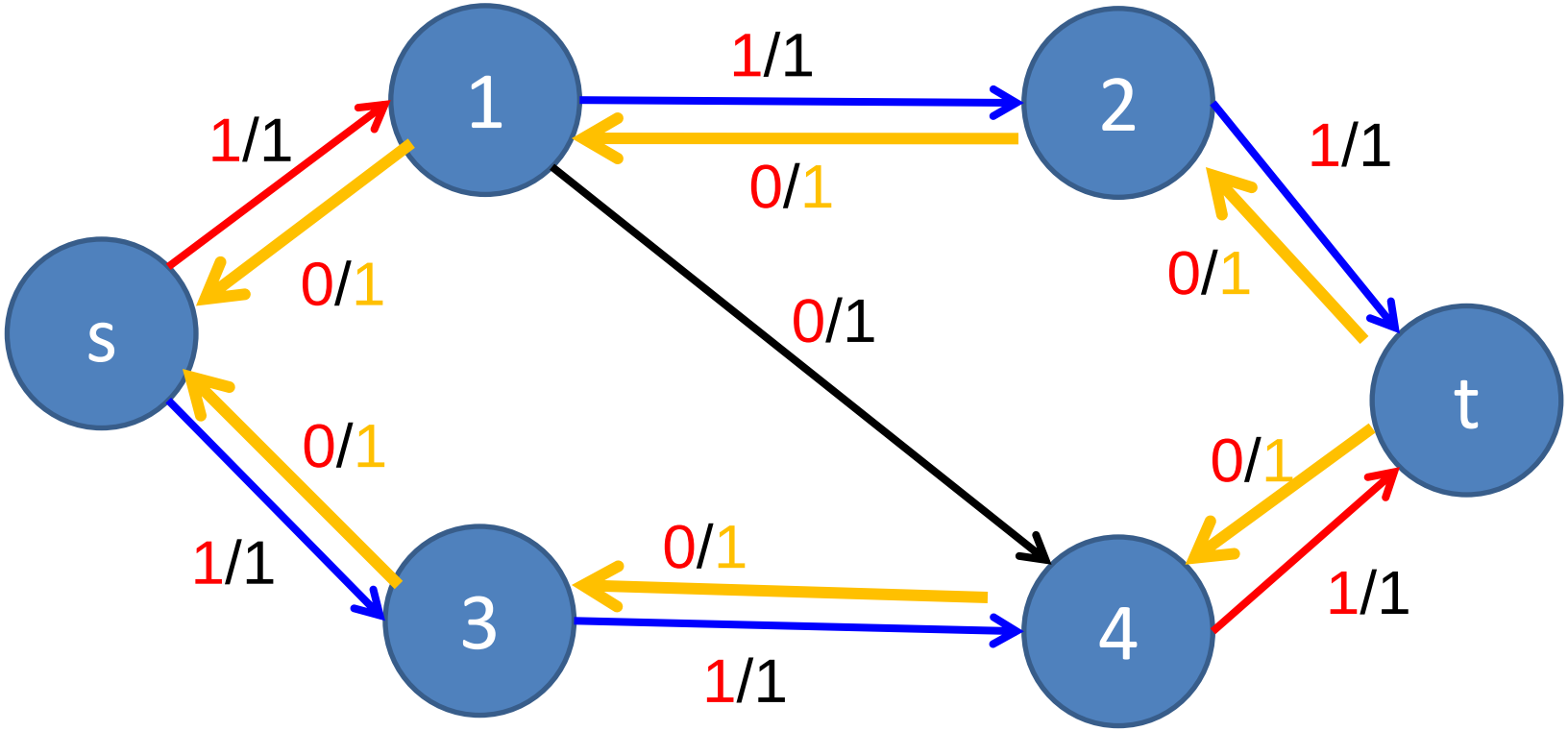


Opposite edge



Maximum flow = 2 !!!

Opposite edge



Maximum flow = 2 !!!

Ford-Fulkerson

1. Find an augmenting path from source to sink
2. Find the bottleneck on augmenting path
3. Let the flow pass
4. Repeat step 1 to step 3 until no augmenting path found

Source Code

```
int FordFulkerson(int n)
{
    int i,j,k;
    int ret=0;

    while(1)
    {
        memset(v,0,sizeof(v));
        if(!DFS(1,n,n)) break;
        ret+=FindFlow(1,n,n);
    }
    return ret;
}
```

Source Code

```
bool DFS(int cur,int t,int n)
{
    int i,j,k;
    v[cur]=1;
    if(cur==t) return true;
    for(i=1;i<=n;i++)
    {
        if(v[i]) continue;

        if(cap[cur][i]-flow[cur][i]>0 || flow[i][cur]>0)
        {
            pa[i]=cur;
            if(DFS(i,t,n)) return true;
        }
    }
    return false;
}
```

Source Code

```
int FindFlow(int s,int t,int n)
{
    int i,j,k,pre;
    int f=INF;

    for(i=t;i!=s;i=pa[i])
    {
        pre=pa[i];

        if(cap[pre][i]-flow[pre][i]>0)
            f=min(f,cap[pre][i]-flow[pre][i]);
        else f=min(f,flow[i][pre]);
    }

    for(i=t;i!=s;i=pa[i])
    {
        pre=pa[i];

        if(cap[pre][i]-flow[pre][i]>0)
            flow[pre][i]+=f;
        else flow[i][pre]-=f;
    }
    return f;
}
```

Maximum Flow

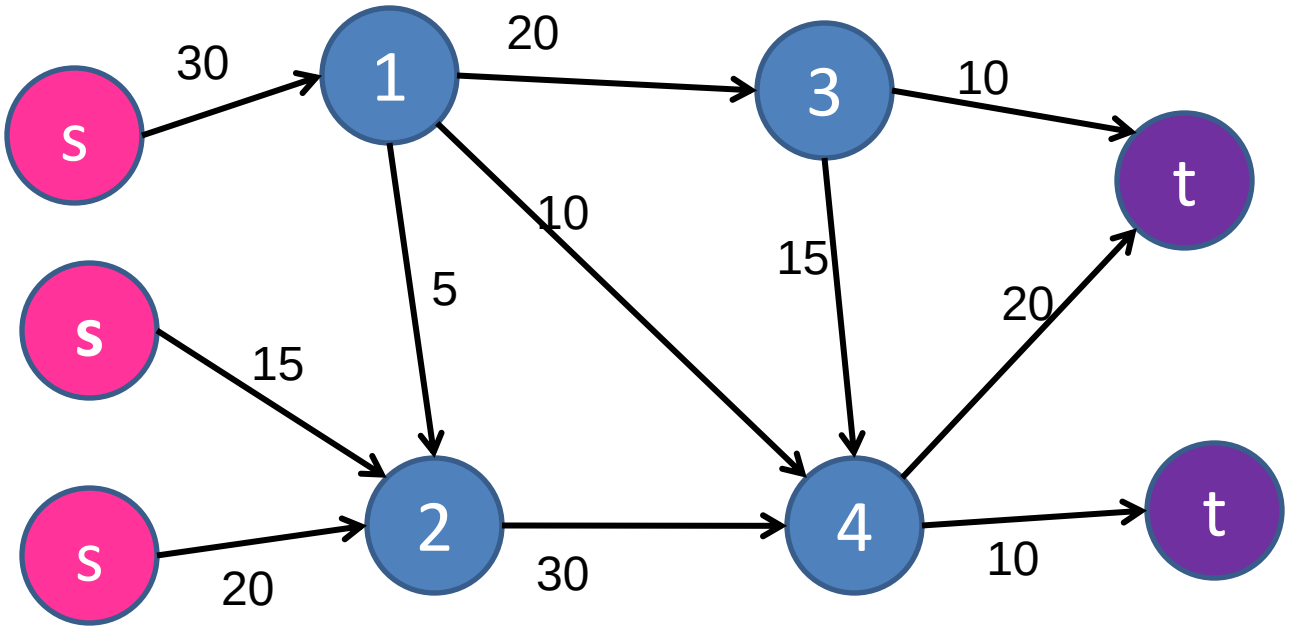
Time complexity: O(EF)

E: number of edges

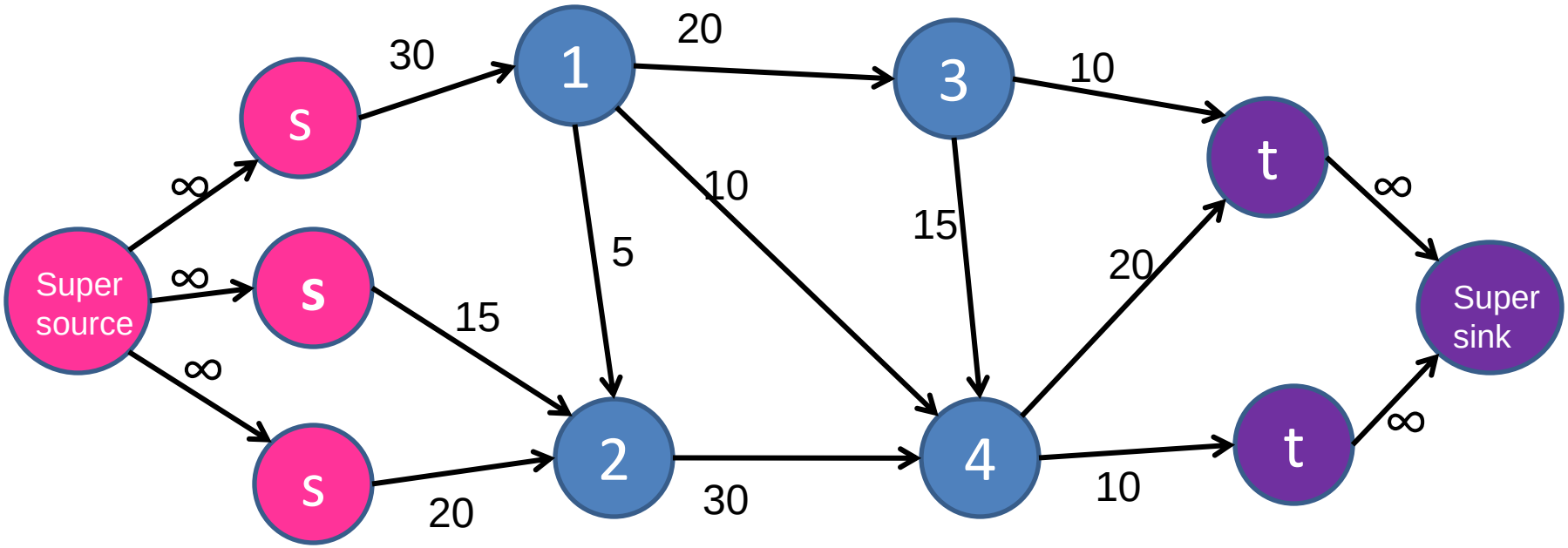
F: value of maximum flow

Learn more ~

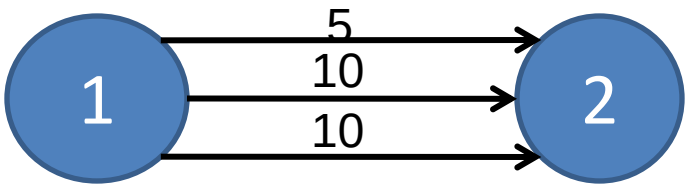
Multi-source & Multi sink



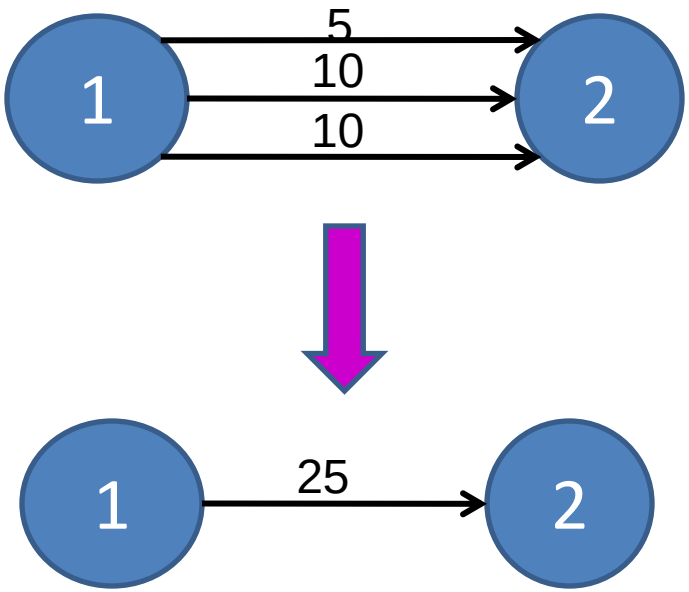
Multi-source & Multi sink



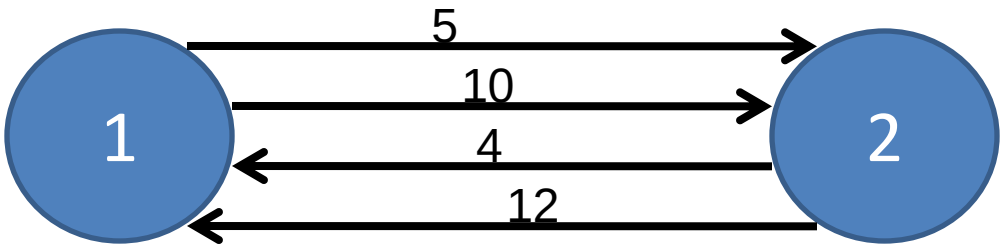
Multi-edge



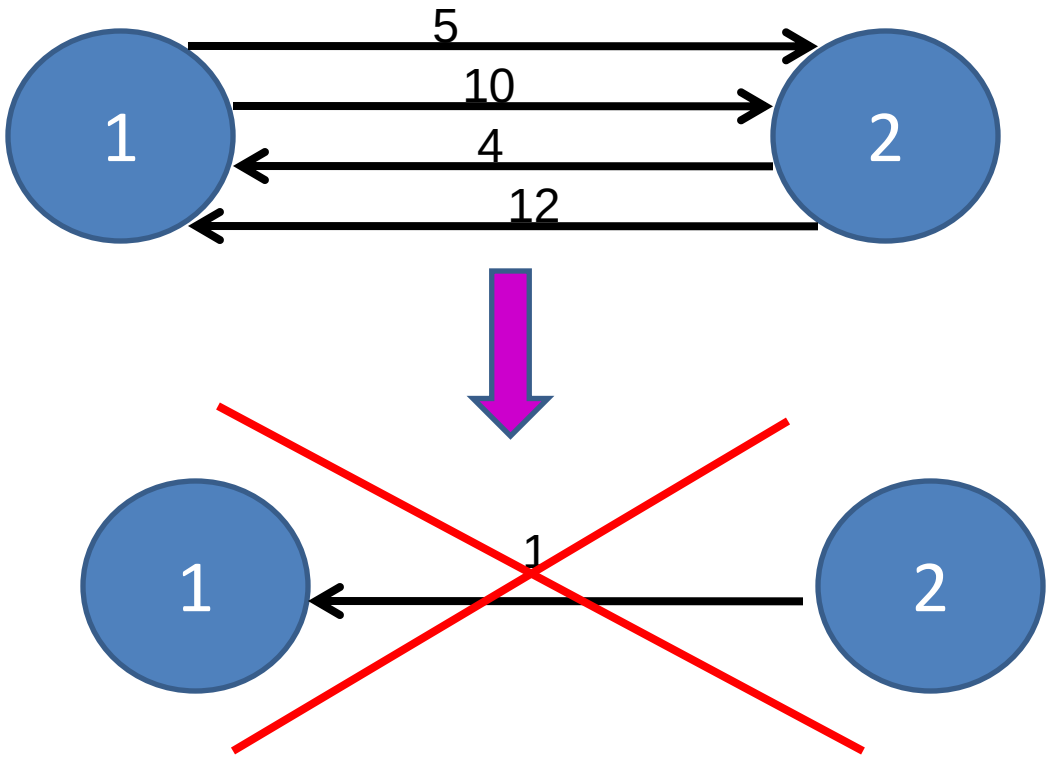
Multi-edge



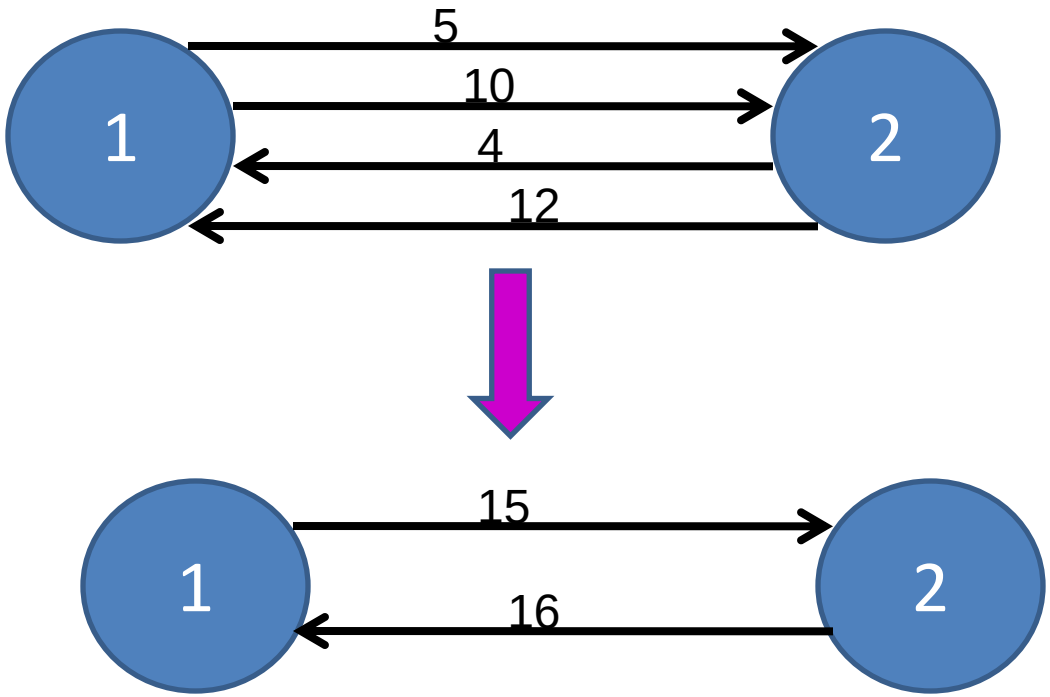
Multi-edge



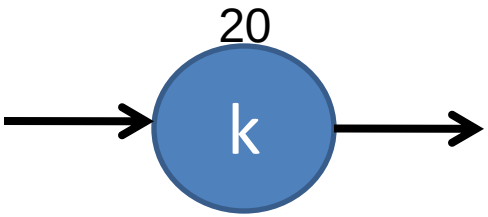
Multi-edge



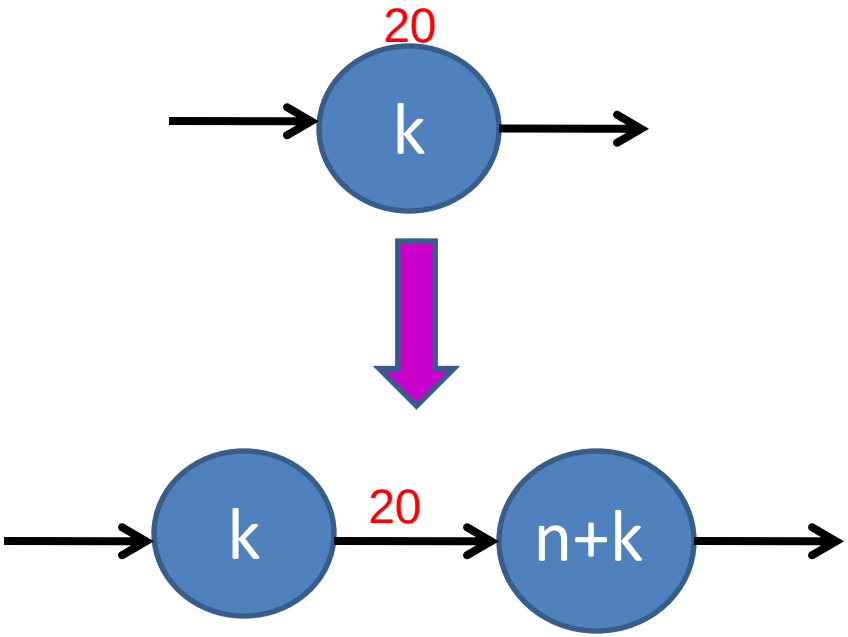
Multi-edge



Capacity on node



Capacity on node



Practice

POJ -2455: Secret Milking Machine

Made By louis7340

Thank you for your attention!