Competitive Algorithm Design and Practice
Minimum Cost Maximum Flow
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Review

Review

- How to reach B from A?
  - DFS/BFS

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- How to reach B from A with minimum cost?
  - Shortest Path Algorithm

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Review

- How to reach B from A?
  - DFS/BFS
- How to reach B from A with minimum cost?
  - Shortest Path Algorithm
- How to reach B from A with maximum flow?
  - Maximum Flow Algorithm
- How to reach B from A with maximum flow, and, with minimum cost?
  - Minimum Cost Maximum Flow


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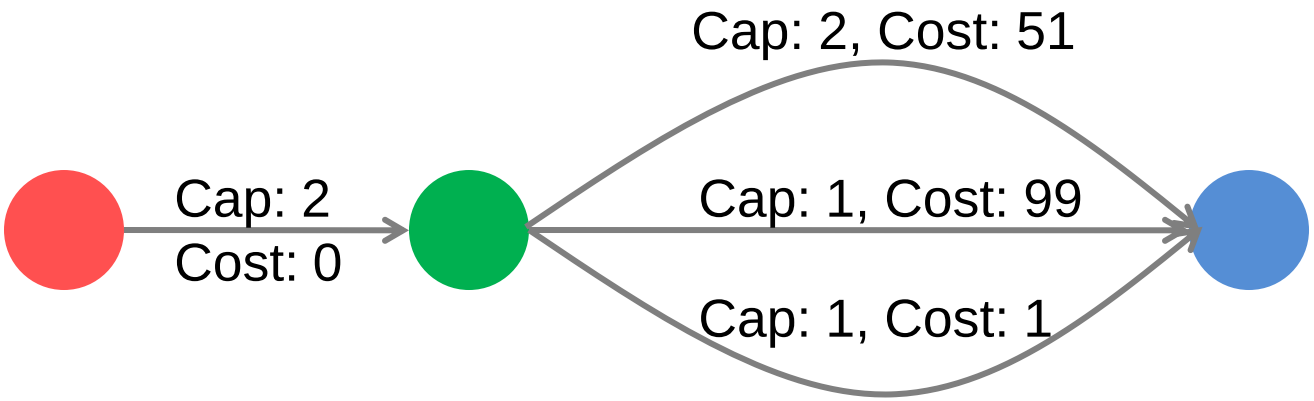
Definition

Definition

- Maximum Flow with Minimum Cost
  - Maximum Flow
  - You should pay k_i (cost/per unit flow) on one $edge_i$



Max Flow: 2
Min Cost: 52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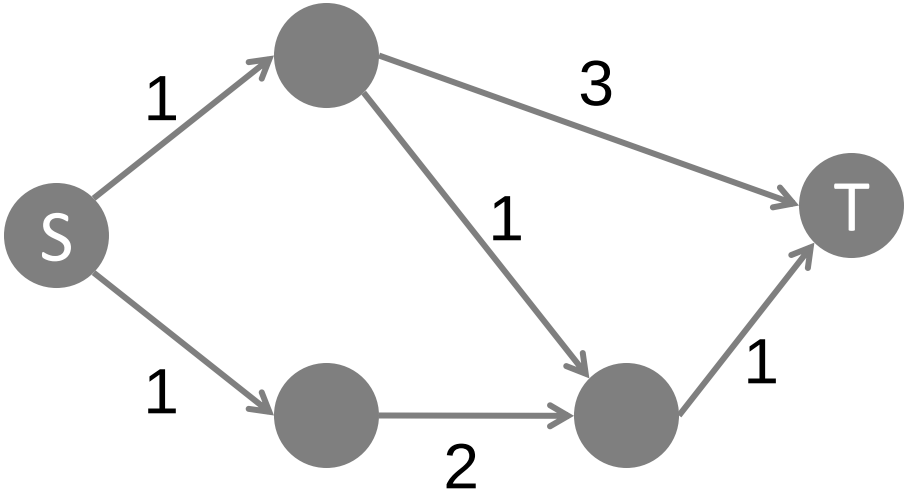
When?

When?

- Scenario 1

  給一張有向圖，兩個人從起點出發，要到達終點，每條邊只能被一人使用，求距離總和的最小值?




```
Min distance:
= 4+4 = 8
```

When?

- Scenario 2

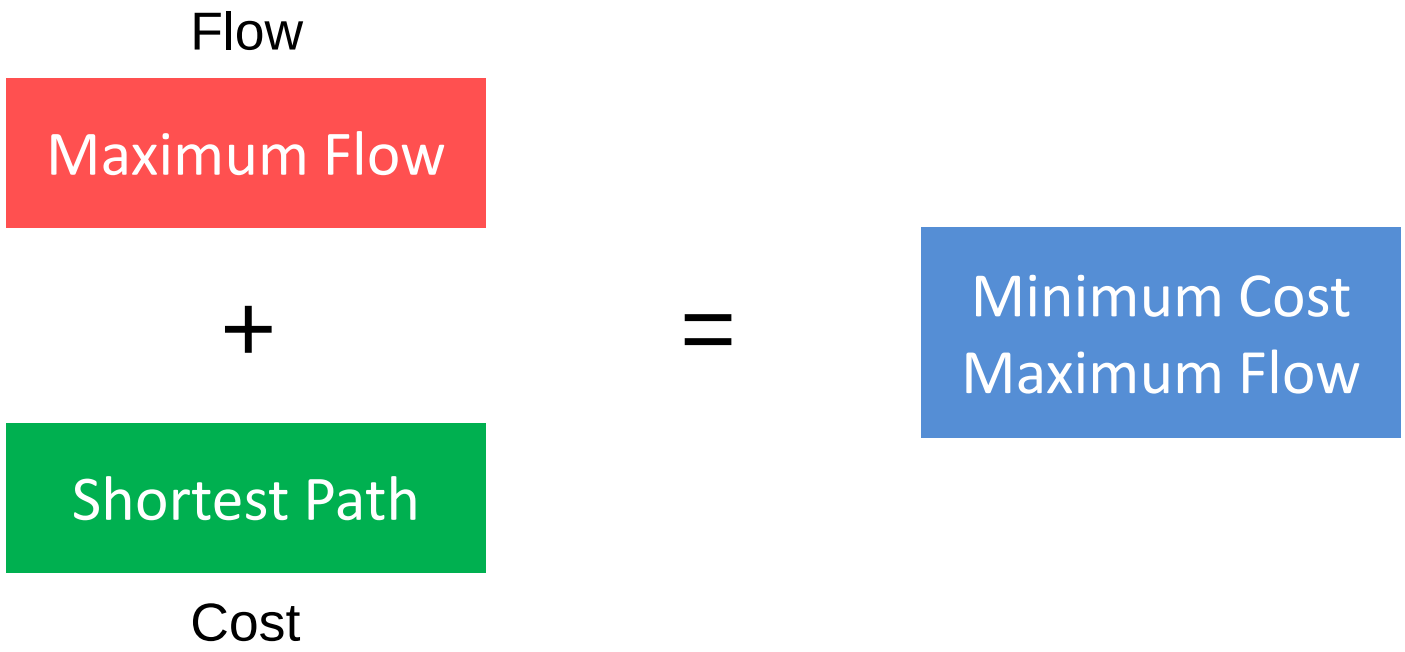
  n個警察，要追n個強盜，根據相對位置的不同，每個警察追達每個強盜的時間不盡相同，假設一個警察要剛好抓到一個強盜，問最少需要多少時間能夠逮捕所有強盜? (1 ≤ n ≤ 100)

When?

- Solution
  - Scenario 1: Minimum Cost Maximum Flow
    - Each edge with capacity = 1, Super source with capacity = 2
  - Scenario 2: Minimum Weight Bipartite Matching
    - Can be modeled with MCMF

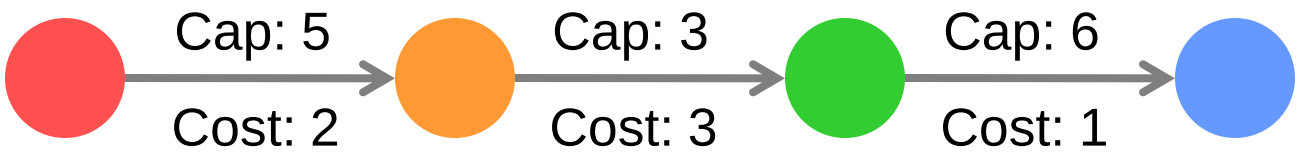
Minimum Cost Maximum Flow

Minimum Cost Maximum Flow



Minimum Cost Maximum Flow

- Cost
  - (Bottleneck flow) x (Total cost along the path)
  - Augmenting Path wit Min. Cost: Shortest Path Algorithm

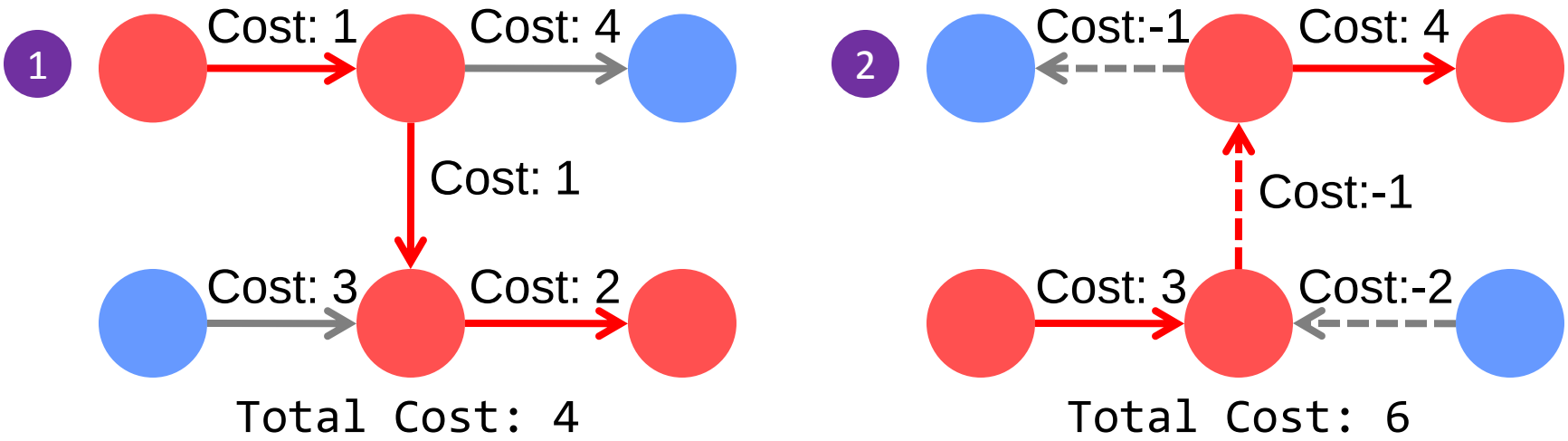


Bottleneck flow: 3 x Min. Cost: 6 = 18

Minimum Cost Maximum Flow

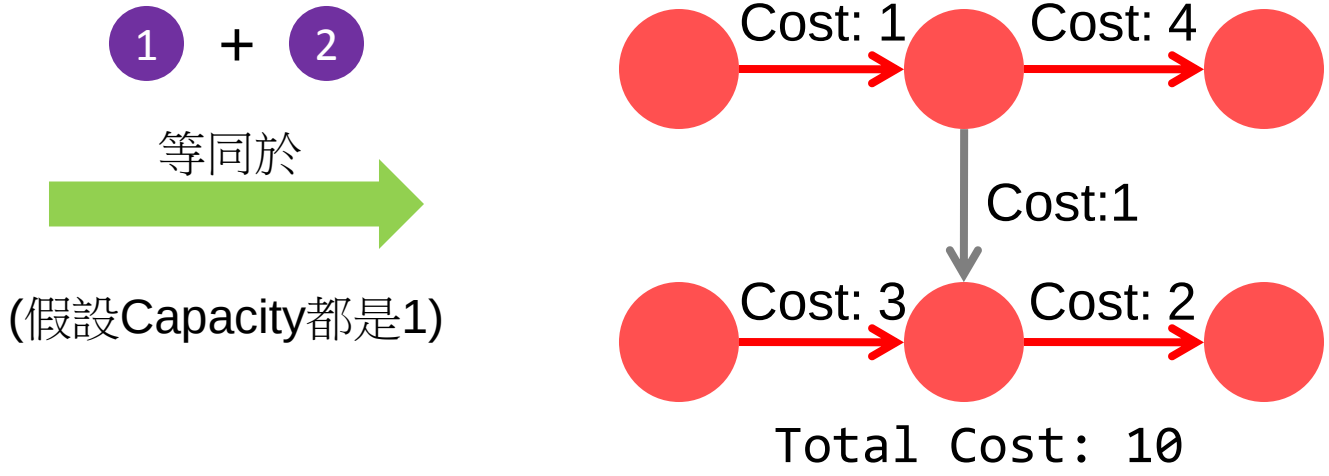
- 逆向流?
  - Cost -> - Cost
  - 反向抵銷Flow，Cost一併抵銷
  - See next slide

Minimum Cost Maximum Flow



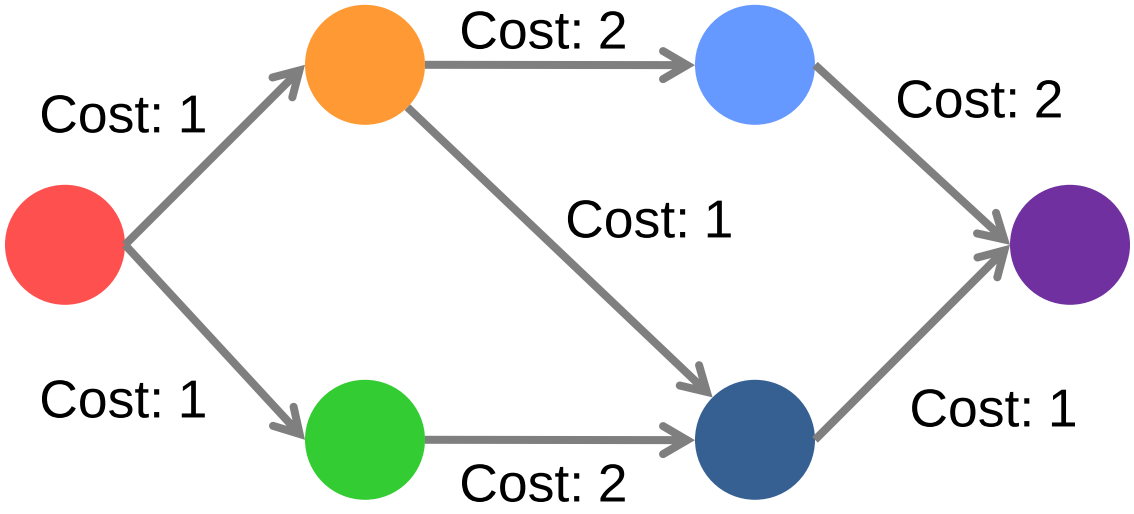
(假設Capacity都是1)

Minimum Cost Maximum Flow



Minimum Cost Maximum Flow

- Example

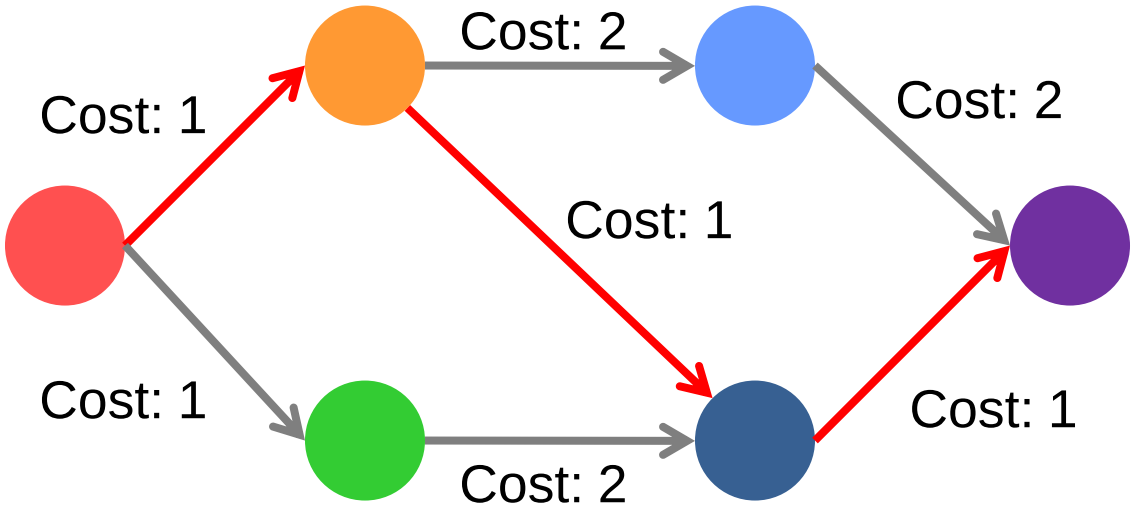


(假設Capacity都是1)

```
Max Flow: 0
Min Cost: 0
```

Minimum Cost Maximum Flow

- Example



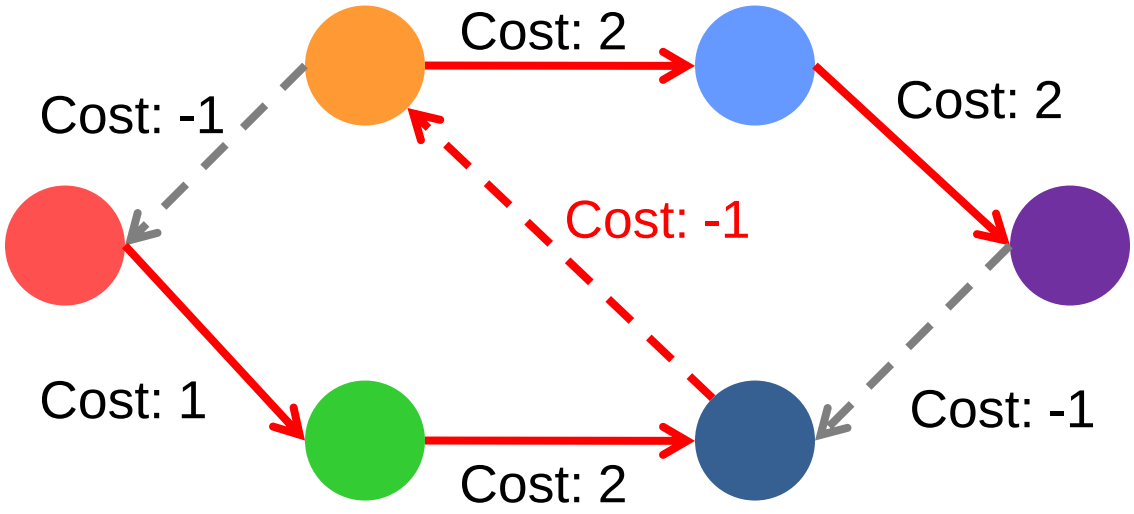
(假設Capacity都是1)

```
Bottleneck: 1
Total Cost: 3

Max Flow: 1
Min Cost: 3
```

Minimum Cost Maximum Flow

- Example



(假設Capacity都是1)

```
Bottleneck: 1
Total Cost: 6

Max Flow: 2
Min Cost: 9
```

Pseudo Code

Pseudo Code

- Pseudo Code

```
1
2   int MCMF(){
3       Max_flow=0
4       Min_cost=0
5       while(SPFA()){
6           ff = min_flow along shortest path;
7           Update the flow along the path;
8           Max_flow += ff;
9           Min_cost += ff * distance of shortest path;
10      }
11      return Max_flow or Min_cost
12  }
```

Pseudo Code

- Pseudo Code

```
1
2   bool SPFA(){
3
4       Initialize;
5       while(Queue is not empty){
6           u = Queue.top();
7           for(each vertex v adjacent to u){
8               if(flow[v][u]>0 and dis[u]-cost[v][u]<dis[v]){
9                   Update distance and record path;
10                  Enqueue if necessary;
11              }
12              else if(cap[u][v]-flow[u][v]>0 and dis[u]+cost[u][v]<dis[v]){
13                  Update distance and record path;
14                  Enqueue if necessary;
15              }
16          }
17      }
18  }
```

Practice

- UVa: 10594
- POJ: 3068

Thank you for your attention!