# Competitive Algorithm Design and Practice
# Longest Common Sub-sequence

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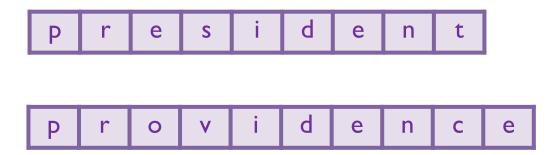Tainan, Taiwan

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# Longest Common Sub-sequence
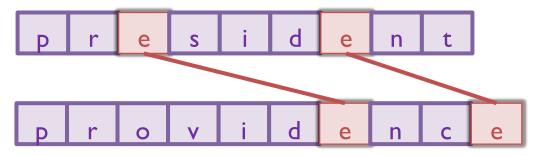
# LCS

- Find a sub-sequence of 2 given sequences in which the sub-sequence`s elements are appear in both original sequences.

- And the sub-sequence is as long as possible.

| p | r | e | s | i | d | e | n | t |
|---|---|---|---|---|---|---|---|---|

| p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|

# LCS

- Common sub-sequence



- Longest one

# LIS

- Naïve solutions:
  - For every sub-sequence in S1, check if it is a sub-sequence of S2.

- Time-complexity:
  - Every sub-sequence, $O(2^N)$, N is length of S1
  - For each sub-sequence checking needs $O(M)$, M is length of S2
  - Total: $O(M*2^N)$

# LCS

- Observation:



- If S1[N] = S2[M], why not include it to LCS?

- Observation:



- If S1[N] ≠ S2[M], it is not possible that both S1[N] and S2[M] are in LCS, at most only one can be.

- As the result, we can exclude either S1[N] or S2[M].

# LCS

- If S1[N] = S2[M],
LCS(S1[1…N],S2[1…M]) =
        LCS(S1[1…N-1],S2[1…M-1])+1


- If S1[N] ≠ S2[M],
LCS(S1[1…N],S2[1…M]) = max(
        LCS(S1[1…N-1],S2[1…M]),
        LCS(S1[1…N],S2[1…M-1])
        )

- What do we want to know?
  - The LCS of S1[1…i] and S2[1…j].
  - i.e. LCS[i][j]

- How can we get that?
  - Find the previous number with longest LIS.
  - LCS[i][j] = {                          , i=0 or j=0
       0
       LCS[i−1][j−1]+1                      , S1[i]=S2[j]
       max(LCS[i−1][j],LCS[i][j−1])  , S1[i]≠S2[j]
    }

# LCS

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | | | | | | | | | | |
| r | 0 | | | | | | | | | | |
| e | 0 | | | | | | | | | | |
| s | 0 | | | | | | | | | | |
| i | 0 | | | | | | | | | | |
| d | 0 | | | | | | | | | | |
| e | 0 | | | | | | | | | | |
| n | 0 | | | | | | | | | | |
| t | 0 | | | | | | | | | | |

# LCS

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | | | | | | | | | | |
| r | 0 | | | | | | | | | | |
| e | 0 | | | | | | | | | | |
| s | 0 | | | | | | | | | | |
| i | 0 | | | | | | | | | | |
| d. | 0 | | | | | | | | | | |
| e. | 0 | | | | | | | | | | |
| n | 0 | | | | | | | | | | |
| t | 0 | | | | | | | | | | |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| P | 0 |   |   |   |   |   |   |   |   |   |   |
| r | 0 |   |   |   |   |   |   |   |   |   |   |
| e | 0 |   |   |   |   |   |   |   |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | l | | | | | | | | | |
| r | 0 | | | | | | | | | | |
| e | 0 | | | | | | | | | | |
| s | 0 | | | | | | | | | | |
| i | 0 | | | | | | | | | | |
| d. | 0 | | | | | | | | | | |
| e. | 0 | | | | | | | | | | |
| n | 0 | | | | | | | | | | |
| t | 0 | | | | | | | | | | |

# LCS

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | | | | | | | | | |
| r | 0 | | | | | | | | | | |
| e | 0 | | | | | | | | | | |
| s | 0 | | | | | | | | | | |
| i | 0 | | | | | | | | | | |
| d. | 0 | | | | | | | | | | |
| e. | 0 | | | | | | | | | | |
| n | 0 | | | | | | | | | | |
| t | 0 | | | | | | | | | | |

# LCS

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | l | | | | | | | | | |
| r | 0 | | | | | | | | | | |
| e | 0 | | | | | | | | | | |
| s | 0 | | | | | | | | | | |
| i | 0 | | | | | | | | | | |
| d. | 0 | | | | | | | | | | |
| e. | 0 | | | | | | | | | | |
| n | 0 | | | | | | | | | | |
| t | 0 | | | | | | | | | | |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | l |   |   |   |   |   |   |   |   |   |
| r | 0 |   |   |   |   |   |   |   |   |   |   |
| e | 0 |   |   |   |   |   |   |   |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | l | l |   |   |   |   |   |   |   |   |
| r | 0 |   |   |   |   |   |   |   |   |   |   |
| e | 0 |   |   |   |   |   |   |   |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | 1 | | | | | | | | |
| r | 0 | | | | | | | | | | |
| e | 0 | | | | | | | | | | |
| s | 0 | | | | | | | | | | |
| i | 0 | | | | | | | | | | |
| d | 0 | | | | | | | | | | |
| e | 0 | | | | | | | | | | |
| n | 0 | | | | | | | | | | |
| t | 0 | | | | | | | | | | |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | 1 | 1 |   |   |   |   |   |   |   |
| r | 0 |   |   |   |   |   |   |   |   |   |   |
| e | 0 |   |   |   |   |   |   |   |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | l | l | l | l |   |   |   |   |   |   |
| r | 0 |   |   |   |   |   |   |   |   |   |   |
| e | 0 |   |   |   |   |   |   |   |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

|    | 0 | p | r | o | v | i | d | e | n | c | e |
|----|---|---|---|---|---|---|---|---|---|---|---|
| 0  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p  | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| r  | 0 | 1 |   |   |   |   |   |   |   |   |   |
| e  | 0 |   |   |   |   |   |   |   |   |   |   |
| s  | 0 |   |   |   |   |   |   |   |   |   |   |
| i  | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n  | 0 |   |   |   |   |   |   |   |   |   |   |
| t  | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| r | 0 | 1 |   |   |   |   |   |   |   |   |   |
| e | 0 |   |   |   |   |   |   |   |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| r | 0 | 1 | 1 |   |   |   |   |   |   |   |   |
| e | 0 |   |   |   |   |   |   |   |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| r | 0 | 1 | 2 |   |   |   |   |   |   |   |   |
| e | 0 |   |   |   |   |   |   |   |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| r | 0 | 1 | 2 |   |   |   |   |   |   |   |   |
| e | 0 |   |   |   |   |   |   |   |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d | 0 |   |   |   |   |   |   |   |   |   |   |
| e | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| r | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| e | 0 | 1 | 2 | 2 | 2 | 2 | 2 | | | | |
| s | 0 | | | | | | | | | | |
| i | 0 | | | | | | | | | | |
| d. | 0 | | | | | | | | | | |
| e. | 0 | | | | | | | | | | |
| n | 0 | | | | | | | | | | |
| t | 0 | | | | | | | | | | |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| r | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| e | 0 | 1 | 2 | 2 | 2 | 2 | 2 |   |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| r | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| e | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |   |   |   |
| s | 0 |   |   |   |   |   |   |   |   |   |   |
| i | 0 |   |   |   |   |   |   |   |   |   |   |
| d. | 0 |   |   |   |   |   |   |   |   |   |   |
| e. | 0 |   |   |   |   |   |   |   |   |   |   |
| n | 0 |   |   |   |   |   |   |   |   |   |   |
| t | 0 |   |   |   |   |   |   |   |   |   |   |

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# LCS

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| r | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| e | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 |
| s | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 |
| i | 0 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 |
| d | 0 | 1 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 4 | 4 |
| e | 0 | 1 | 2 | 2 | 2 | 3 | 4 | 5 | 5 | 5 | 5 |
| n | 0 | 1 | 2 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 6 |
| t | 0 | 1 | 2 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 6 |

# Uva_531

# Learn more!

- Sometimes we need to <span style="color:red">output</span> a <span style="color:red">solution</span> too, but <span style="color:green">how</span>?

- Additional array pre[][].
  - pre[i][j] =
    ↘ if S1[i]=S2[j]
    ↑ if S1[i]≠S2[j] && LCS[i][j]=LCS[i-1][j]
    ← if S1[i]≠S2[j] && LCS[i][j]=LCS[i][j-1]

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← |
| t | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← |   |
| t | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ← |
| t | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← |
| t | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↗ | ← | ← |
| t | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← |
| t | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↗ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↗ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↗ | ← | ← |
| t | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

|   | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← |
| t |   | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

# Learn more!

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← |
| t | | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← |
| t | | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↗ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↗ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↗ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↗ | ← | ← |
| t | | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← |
| t | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!

| | 0 | p | r | o | v | i | d | e | n | c | e |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| p | 0 | ↖ | ← | ← | ← | ← | ← | ← | ← | ← | ← |
| r | 0 | ↑ | ↖ | ← | ← | ← | ← | ← | ← | ← | ← |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| s | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| i | 0 | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ↑ | ↑ | ↑ | ↑ |
| d | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ↑ | ↑ | ↑ | ↑ |
| e | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← | ↖ |
| n | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↖ | ← | ← |
| t | 0 | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |

# Learn more!