

NCKU Programming Contest Training Course

二維DP

2018/05/02

Jheng-Huang Hong

Department of Computer Science and Information Engineering
National Cheng Kung University
Tainan, Taiwan



Outline

- Coin problem
- MSS problem



Coin change

- Uva 10306
- Coin 有 X,Y 兩種價值
- 求最少需多少 Coin 能使 $\sqrt{X^2 + Y^2} =$ 目標值

Sample Input

```
3
{ 2 5
  0 2
  2 0
  3 20
  0 2
  2 0
  2 1
  3 5
  3 0
  0 4
  5 5
```

Sample Output

```
not possible
10
2
```



Coin change

- Example:
- Input:
 - 1
 - 2 10
 - 3 1
 - 0 3
- Output:
 - 4



Coin change

- 硬幣無限
- $dp[i][j]$: 湊到 $X=i, Y=j$ 所需的最少硬幣數
- $dp[0][0]=0$, 其餘為 Inf
- Inf : 目前湊不到
- 當 $X = 3, Y = 1$ 時 (next page)
- $for(i=3;i \leq 10; ++i)$
 $for(j=1;j \leq 10; ++j)$
 $dp[i][j] = \min(dp[i][j], dp[i-3][j-1]+1)$



X



International Collegiate Programming Contest

event sponsor

Y

	0	1	2	3	4	5	6	7	8	9	10
0	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
1	Inf	Inf	Inf	1	Inf	Inf	Inf	Inf	Inf	Inf	Inf
2	Inf	Inf	Inf	Inf	Inf	Inf	2	Inf	Inf	Inf	Inf
3	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	3	Inf
4	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
5	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
6	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
7	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
8	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
9	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
10	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf

NCKU



Coin change

- 硬幣無限
- $dp[i][j]$: 湊到 $X=i, Y=j$ 所需的最少硬幣數
- $dp[0][0]=0$, 其餘為 Inf
- Inf : 目前湊不到
- 當 $X = 0, Y = 3$ 時 (next page)
- $for(i=0; i \leq 10; ++i)$
 $for(j=3; j \leq 10; ++j)$
 $dp[i][j] = \min(dp[i][j], dp[i-0][j-3]+1)$



X



International Collegiate Programming Contest

event sponsor

Y

	0	1	2	3	4	5	6	7	8	9	10
0	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
1	Inf	Inf	Inf	1	Inf	Inf	Inf	Inf	Inf	Inf	Inf
2	Inf	Inf	Inf	Inf	Inf	Inf	2	Inf	Inf	Inf	Inf
3	1	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	3	Inf
4	Inf	Inf	Inf	2	Inf	Inf	Inf	Inf	Inf	Inf	Inf
5	Inf	Inf	Inf	Inf	Inf	Inf	3	Inf	Inf	Inf	Inf
6	2	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	4	Inf
7	Inf	Inf	Inf	3	Inf	Inf	Inf	Inf	Inf	Inf	Inf
8	Inf	Inf	Inf	Inf	Inf	Inf	4	Inf	Inf	Inf	Inf
9	3	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	5	Inf
10	Inf	Inf	Inf	4	Inf	Inf	Inf	Inf	Inf	Inf	Inf

NCKU



X



International Collegiate Programming Contest

event sponsor

Y

	0	1	2	3	4	5	6	7	8	9	10
0	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
1	Inf	Inf	Inf	1	Inf	Inf	Inf	Inf	Inf	Inf	Inf
2	Inf	Inf	Inf	Inf	Inf	Inf	2	Inf	Inf	Inf	Inf
3	1	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	3	Inf
4	Inf	Inf	Inf	2	Inf	Inf	Inf	Inf	Inf	Inf	Inf
5	Inf	Inf	Inf	Inf	Inf	Inf	3	Inf	Inf	Inf	Inf
6	2	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	4	Inf
7	Inf	Inf	Inf	3	Inf	Inf	Inf	Inf	Inf	Inf	Inf
8	Inf	Inf	Inf	Inf	Inf	Inf	4	Inf	Inf	Inf	Inf
9	3	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	5	Inf
10	Inf	Inf	Inf	4	Inf	Inf	Inf	Inf	Inf	Inf	Inf

NCKU



Maximum Segment Sum

- Uva 108
- 求最大矩形區塊和

0	-2	-7	0
9	2	-6	2
-4	1	-4	1
-1	8	0	-2

$$9+2-4+1-1+8 = 15$$



Maximum Segment Sum

- $dp[i][j]$: $array[i][0] + array[i][1] + \dots + array[i][j]$

Input array

0	-2	-7	0
9	2	-6	2
-4	1	-4	1
-1	8	0	-2

dp

0	-2	-9	-9
9	11	5	7
-4	-3	-7	-6
-1	7	7	5



Maximum Segment Sum

Input array

0	-2	-7	0
9	2	-6	2
-4	1	-4	1
-1	8	0	-2

=

dp

0	-2	-9	-9
9	11	5	7
-4	-3	-7	-6
-1	7	7	5

$$dp[1][1] + dp[2][1] + dp[3][1]$$



Maximum Segment Sum

需要 $O(n^4)$ 以內演算法



Thank for Your Attention

