NCKU Programming Contest Training Course
Math
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Outline

Prime Numbers

Big Number

GCD, Extended Euclid's Algorithm

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Prime Number

- Sieve of Eratosthenes (埃拉托斯特尼篩法)
  - 由小到大選擇質數，並刪除其倍數
- 6n±1 Method
  - 拿 2 和 3 這兩個質數先篩過一遍，剩下的數字則用除法驗證是不是質數。

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Prime Number

- We use **sieve** to create a prime array
  - Chose the smallest number at each iteration and delete the multiple of this number

Chose 2

2 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ 9 ~~10~~ 11 ~~12~~ 13 ~~14~~ 15 ~~16~~ 17 ~~18~~ 19 ~~20~~ 21 ~~22~~…

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Prime Number

- We use **sieve** to create a prime array
  - Chose the smallest number at each iteration and delete the multiple of this number

Chose 3

2 ~~3~~ ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~ 11 ~~12~~ 13 ~~14~~ ~~15~~ ~~16~~ 17 ~~18~~ 19 ~~20~~ ~~21~~ ~~22~~...

Prime Number

- We use **sieve** to create a prime array
  - Chose the smallest number at each iteration and delete the multiple of this number

Chose 5

2 ~~3~~ ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~ 11 ~~12~~ 13 ~~14~~ ~~15~~ ~~16~~ 17 ~~18~~ 19 ~~20~~ ~~21~~ ~~22~~…

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Prime Number

- We use **sieve** to create a prime array
  - Chose the smallest number at each iteration and delete the multiple of this number

Chose 7

2 ~~3~~ ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~ 11 ~~12~~ 13 ~~14~~ ~~15~~ ~~16~~ 17 ~~18~~ 19 ~~20~~ ~~21~~ ~~22~~…

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Prime Number

- We use **sieve** to create a prime array
  - Chose the smallest number at each iteration and delete the multiple of this number

Chose 11

2 ~~3~~ ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~ 11 ~~12~~ 13 ~~14~~ ~~15~~ ~~16~~ 17 ~~18~~ 19 ~~20~~ ~~21~~ ~~22~~…

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Prime Number

- Sieve of Eratosthenes (埃拉托斯特尼篩法)
  - 由小到大選擇質數，並刪除其倍數

```
#include <cmath>
#include <cstring>
#define MAX 10000000
bool is_prime[MAX];
void eratosthenes()
{
    memset(is_prime, 1, sizeof(is_prime));
    is_prime[0] = false;
    is_prime[1] = false;

    for (int i = 2; i <= sqrt(MAX); ++i)
        if (is_prime[i])
            for(int j = i+i; j < MAX; j += i)
                is_prime[j] = false;
}
```


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Prime Number

- 6n±1 Method
  - – 2 和 3 的最小公倍數是 6 ，把所有數字分為 6n 、 6n+1 、 6n+2 、
  - 6n+3 、 6n+4 、 6n+5 六種，可以看出 6n 、 6n+2 、 6n+3 、 6n+4 會是 2 或 3 的倍數，不屬於質數。因此，只要驗證 6n+1 和 6n+5 (= 6n-1)是不是質數就可以了。

Prime Number

- 6n±1 Method

```
#include <vector>
#define MAX 10000000
vector<int> prime;
bool is_prime(int n) {
    for (int i = 0; prime[i]*prime[i] <= n; ++i)
        if (n % prime[i] == 0)
            return false;
    return true;
}
void make_prime() {
    prime.push_back(2);
    prime.push_back(3);
    for (int i = 5, gap = 2; i < MAX; i+=gap, gap = 6 - gap)
        if (is_prime(i))
            prime.push_back(i);
}
```

Prime Number

- 方法二比方法一慢，但較省空間
- But just remember that the code in previous page is fast enough to solve almost every prime problems

- 其他方法:
  - [演算法 筆記 - Prime]

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Practice - 1

UVa 10392 - Factoring Large Numbers

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Outline

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Big Number

- Array
- 習慣上將低位數放在index比較小的位置
  - Ex:680468975231245

0	1	2	3	4	5	6	7	8	9	…								
5	4	2	1	3	2	5	7	9	8	6	4	0	8	6	_	_	_	- - - -

- 右方補0

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Big Number

- 加法：位數各自相加後，由低至高位依序進位
- 減法：位數各自相減後，由低至高位依序借位
- 乘法：直式乘法
- 除法：長除法

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Big Number

- 加法：

```
void add(int a[100], int b[100], int c[100]) {
    for (int i = 0; i < 100; ++i)
        c[i] = a[i] + b[i];

    for (int i = 0; i < 100-1; ++i) {
        c[i+1] += c[i] / 10;
        c[i] %= 10;
    }
}
```

Practice - 2

UVa 10106 - Product

Problem Description

The problem is to multiply two integers X,Y .(0 ≤ X,Y < 10250)

Input

The input will consist of a set of pairs of lines. Each line in pair contains one multiplyer.

Output

For each input pair of lines the output line should consist one integer the product.

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Outline

Prime Numbers

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GCD, Extended Euclid's Algorithm

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Greatest Common Divisor

- 輾轉相除法 (Euclidean Algorithm)

```
int gcd(int a, int b) {
    if (a == 0)
        return b;
    return gcd(b % a, a);
}
```

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- gcd(462, 1071)
  - gcd(147, 462)
    - gcd(21, 147)
      - gcd(0, 7)

```
int gcd(int a, int b) {
    if (a == 0)

    r
}
```

- 從1071中不斷減去462直到小於462（可以減2次，即商q0 = 2），餘數是147：
  - 1071 = 2 x 462 + 147.
- 然後從462中不斷減去147直到小於147（可以減3次，即q1 = 3），餘數是21：
  - 462 = 3 x 147 + 21.
- 再從147中不斷減去21直到小於21（可以減7次，即q2 = 7），沒有餘數：
  - 147 = 7 x 21 + 0.
- 此時，餘數是0，所以1071和462的最大公因數是21，

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Practice - 3

UVa 408 – Uniform Generator

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Extended Euclidean Algorithm

- 找到$aX + bY = gcd(a,b)$的整數解 X,Y
- Ex (from wiki)
  - $47x + 30y = 1$

Extended Euclidean Algorithm

- 47 = 30 * 1 + 17
- 30 = 17 * 1 + 13
- 17 = 13 * 1 + 4
- 13 = 4 * 3 + 1
- 4 = 1 * 4 + 0

gcd(30, 47)
gcd(17, 30)
gcd(13, 17)
gcd(4, 13)
gcd(1, 4)
gcd(0, 1)

Extended Euclidean Algorithm

- $47 = 30 * 1 + 17$
- $30 = 17 * 1 + 13$
- $17 = 13 * 1 + 4$
- $13 = 4 * 3 + 1$
- $4 = 1 * 4 + 0$

- $17 = 47 * 1 + 30 * (-1)$
- $13 = 30 * 1 + 17 * (-1)$
- $4 = 17 * 1 + 13 * (-1)$
- $1 = 13 * 1 + 4 * (-3)$

$47x + 30y = 1$

Extended Euclidean Algorithm

- $1 = 13 * 1 + 4 * (-3)$
- $1 = 13 * 1 + [17 * 1 + 13 * (-1)] * (-3)$
- $1 = 17 * (-3) + 13 * 4$
- $1 = 17 * (-3) + [30 * 1 + 17 * (-1)] * 4$
- $1 = 30 * 4 + 17 * (-7)$
- $1 = 30 * 4 + [47 * 1 + 30 * (-1)] * (-7)$
- $1 = 47 * (-7) + 30 * 11$

Extended Euclidean Algorithm

- $gcd(a,b) = gcd(b,a\%b)$
- $aX + bY = gcd(a,b) = gcd(b,a\%b) = bX' + (a\%b)Y'$
- $aX + bY = bX' + [a - (a/b)b]Y' = aY' + b(X'-(a/b)Y')$
  - $X = Y'$
  - $Y = X' - (a/b)Y'$

Extended Euclidean Algorithm

```
int exGCD(int a, int b, int &X, int &Y) {
    if (b == 0) {
        X = 1;
        Y = 0;
        return a;
    } else {
        int gcd = exGCD(b, a % b, X, Y);
        int tmp = X;
        X = Y;
        Y = tmp - (a/b)*Y;
        return gcd;
    }
}
```

Practice - 4

UVa 10104 - Euclid Problem

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Extended Euclidean Algorithm

- $\frac{m!}{n!}$ % P (P是一個很大的質數)

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Extended Euclidean Algorithm

- $aX + bY = \gcd(a, b)$
  - $a = n!$
  - $b = p$
  - $\gcd(a,b) = 1$

方程式 ax+by=1 有整數解
iff 整數 a 和 b 互質

Extended Euclidean Algorithm

- n!X + pY = 1 (use Extended Euclidean Algorithm get (X,Y))
- n!X + pY = 1 $\rightarrow$ mod p
- (n!X) % p = 1 --- (1)
- $\frac{m!}{n!} \% \, p = ans$ --- (2)
- (1) * (2)

$$\rightarrow \left(\frac{m!}{n!} \times n!\, X \right) \% \, p = ans \;\rightarrow (\, m! \times X \,) \, \% \, p = ans$$

Practice - 5

Facebook Hacker Cup 2017 Round1

Beach Umbrellas

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Epsilon ε

- Float :
  - 數值範圍 :-3.4e-38 ~ 3.4e38
  - 十位數精確度位數 :6~7
- Double :
  - 數值範圍 :-1.7e308~1.7e308
  - 十位數精確度位數 :14~15

Epsilon ε

- Example

```
#include <cstdio>
#include <cmath>

int main() {
    double a = asin(sqrt(2.0) / 2) * 4.0;
    double b = acos(-1.0);

    printf("a = %.20lf\n", a);
    printf("b = %.20lf\n", b);
    printf("a-b = %.20lf\n", a - b);
    printf("a == b? %s\n", a == b ? "True" : "False");
}
```

Epsilon ε

- Result

```
linyunwen@Lin-Yun-Wens-MacBook-Air ~/D/L/c/ACM> ./sample_epsilon
a = 3.14159265358979356009
b = 3.14159265358979311600
a-b = 0.00000000000000044409
a == b? False
```

Epsilon ε

- 引入 eps 判斷浮點數是否相等
  - eps = 1e-8

整數	浮點數
a == b	$\lvert a - b \rvert < eps$
a != b	$\lvert a - b \rvert > eps$
a < b	$a - b < -eps$
a > b	$a - b > eps$

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Practice - 6

UVa 906 – Rational Neighbor

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