

Competitive Algorithm Design and Practice

Longest Common Sub-sequence

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http://myweb.ncku.edu.tw/~f74991073/20140319_DP.zip

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Longest Common Sub-sequence



LCS

- Find a **sub-sequence** of 2 given sequences in which the **sub-sequence's elements** appear in both original sequences.
- And the sub-sequence is **as long as possible**.

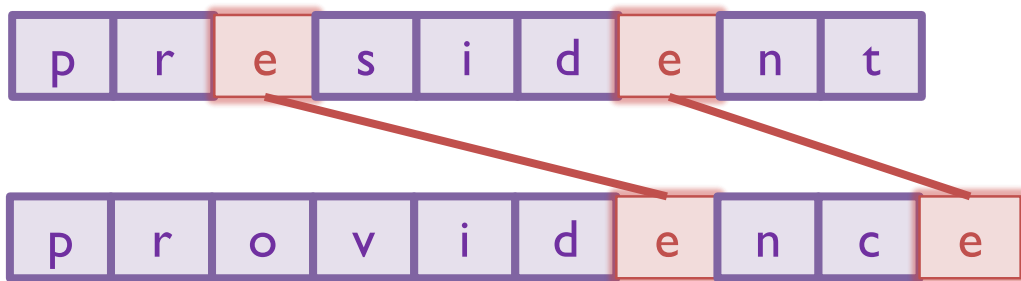
p	r	e	s	i	d	e	n	t
---	---	---	---	---	---	---	---	---

p	r	o	v	i	d	e	n	c	e
---	---	---	---	---	---	---	---	---	---



LCS

- Common sub-sequence



- Longest one



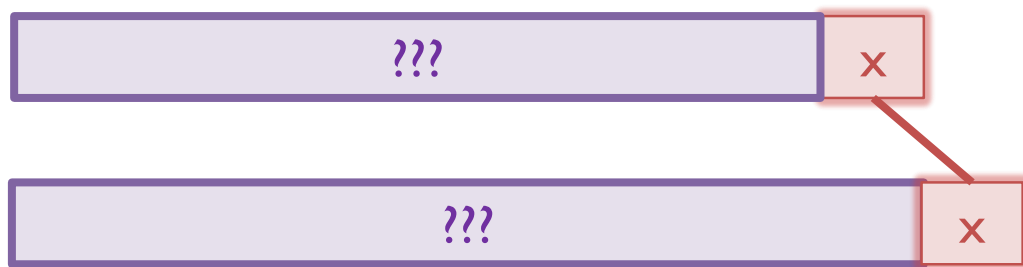
LIS

- Naïve solutions:
 - For every sub-sequence in $s1$, check if it is a sub-sequence of $s2$.
- Time-complexity:
 - Every sub-sequence, $O(2^N)$, N is length of $s1$
 - For each sub-sequence checking needs $O(M)$,
 M is length of $s2$
 - Total: $O(M * 2^N)$



LCS

- Observation:

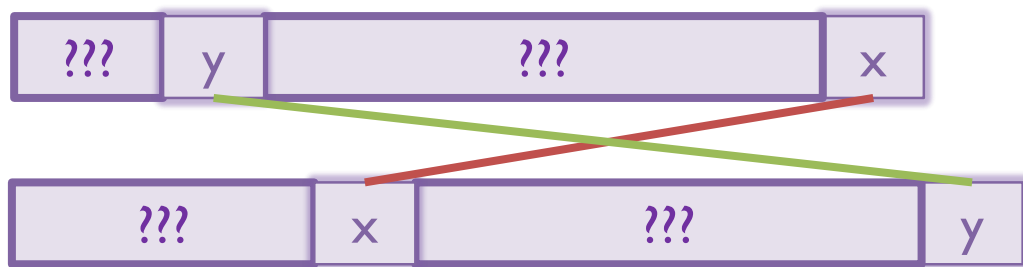


- If $s1[N] = s2[M]$, why not include it to LCS?



LCS

- Observation:



- If $s1[N] \neq s2[M]$, it is not possible that both $s1[N]$ and $s2[M]$ are in LCS, at most only one can be.
- As the result, we can exclude either $s1[N]$ or $s2[M]$.



LCS

- If $s1[N] = s2[M]$,
$$\text{LCS}(s1[1\dots N], s2[1\dots M]) = \text{LCS}(s1[1\dots N-1], s2[1\dots M-1]) + 1$$
- If $s1[N] \neq s2[M]$,
$$\text{LCS}(s1[1\dots N], s2[1\dots M]) = \max(\text{LCS}(s1[1\dots N-1], s2[1\dots M]), \text{LCS}(s1[1\dots N], s2[1\dots M-1]))$$



LCS

- what do we want to know?
 - The LCS of $s1[1..i]$ and $s2[1..j]$.
 - i.e. $LCS[i][j]$
- How can we get that?
 - Find the previous number with longest LIS.
 - $LCS[i][j] = \{$
 - 0 , $i=0$ or $j=0$
 - $LCS[i-1][j-1]+1$, $s1[i]=s2[j]$
 - $\max(LCS[i-1][j], LCS[i][j-1])$, $s1[i] \neq s2[j]$
 - $\}$



LCS

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0										
r	0										
e	0										
s	0										
i	0										
d	0										
e	0										
n	0										
t	0										



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0											
r	0											
e	0											
s	0											
i	0											
d	0											
e	0											
n	0											
t	0											



LCS

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0										
r	0										
e	0										
s	0										
i	0										
d	0										
e	0										
n	0										
t	0										



LCS

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	l									
r	0										
e	0										
s	0										
i	0										
d	0										
e	0										
n	0										
t	0										



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	l										
r	0											
e	0											
s	0											
i	0											
d	0											
e	0											
n	0											
t	0											



LCS

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	l									
r	0										
e	0										
s	0										
i	0										
d	0										
e	0										
n	0										
t	0										



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	l										
r	0											
e	0											
s	0											
i	0											
d	0											
e	0											
n	0											
t	0											



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	0	l	l								
r	0											
e	0											
s	0											
i	0											
d	0											
e	0											
n	0											
t	0											



LCS

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	l	l								
r	0										
e	0										
s	0										
i	0										
d	0										
e	0										
n	0										
t	0										



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	l	l	l								
r	0											
e	0											
s	0											
i	0											
d	0											
e	0											
n	0											
t	0											



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	0	1	1	1	1						
r	0											
e	0											
s	0											
i	0											
d	0											
e	0											
n	0											
t	0											



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	l	l	l	l	l	l	l	l	l	l	l
r	0	l										
e	0											
s	0											
i	0											
d	0											
e	0											
n	0											
t	0											



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	l	l	l	l	l	l	l	l	l	l	l
r	0	l										
e	0											
s	0											
i	0											
d	0											
e	0											
n	0											
t	0											



LCS

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	1	1	1	1	1	1	1	1	1	1
r	0	1									
e	0										
s	0										
i	0										
d	0										
e	0										
n	0										
t	0										



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	0	1	1	1	1	1	1	1	1	1	1
r	0	0	1	2								
e	0											
s	0											
i	0											
d	0											
e	0											
n	0											
t	0											



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	0	1	1	1	1	1	1	1	1	1	1
r	0	0	1	2								
e	0	0										
s	0	0										
i	0	0										
d	0	0										
e	0	0										
n	0	0										
t	0	0										



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	0	1	1	1	1	1	1	1	1	1	1
r	0	0	1	2	2	2	2	2	2	2	2	2
e	0	0	1	2	2	2	2	2				
s	0	0										
i	0	0										
d	0	0										
e	0	0										
n	0	0										
t	0	0										



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	0	1	1	1	1	1	1	1	1	1	1
r	0	0	1	2	2	2	2	2	2	2	2	2
e	0	0	1	2	2	2	2	2				
s	0	0										
i	0	0										
d	0	0										
e	0	0										
n	0	0										
t	0	0										



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	0	1	1	1	1	1	1	1	1	1	1
r	0	0	1	2	2	2	2	2	2	2	2	2
e	0	0	1	2	2	2	2	2	3			
s	0	0										
i	0	0										
d	0	0										
e	0	0										
n	0	0										
t	0	0										



LCS

		0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0
p	0	0	1	1	1	1	1	1	1	1	1	1
r	0	0	1	2	2	2	2	2	2	2	2	2
e	0	0	1	2	2	2	2	2	3	3	3	3
s	0	0	1	2	2	2	2	2	3	3	3	3
i	0	0	1	2	2	2	3	3	3	3	3	3
d	0	0	1	2	2	2	3	4	4	4	4	4
e	0	0	1	2	2	2	3	4	5	5	5	5
n	0	0	1	2	2	2	3	4	5	6	6	6
t	0	0	1	2	2	2	3	4	5	6	6	6



Uva 531



Learn more!

- Sometimes we need to **output** a **solution** too, but **how**?
- Additional array **pre**[][].
 - $\text{pre}[i][j] =$
 - \nwarrow if $s1[i]=s2[j]$
 - \uparrow if $s1[i]\neq s2[j] \ \&\& \ \text{LCS}[i][j]=\text{LCS}[i-1][j]$
 - \leftarrow if $s1[i]\neq s2[j] \ \&\& \ \text{LCS}[i][j]=\text{LCS}[i][j-1]$



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑



Learn more!

	0	p	r	o	v	i	d	e	n	c	e
0	0	0	0	0	0	0	0	0	0	0	0
p	0	↖	←	←	←	←	←	←	←	←	←
r	0	↑	↖	←	←	←	←	←	←	←	←
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
s	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
i	0	↑	↑	↑	↑	↖	←	↑	↑	↑	↑
d	0	↑	↑	↑	↑	↑	↖	↑	↑	↑	↑
e	0	↑	↑	↑	↑	↑	↑	↖	←	←	↖
n	0	↑	↑	↑	↑	↑	↑	↑	↖	←	←
t	0	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

